Department of Mechanical Engineering

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LECTURE NOTES

Name of the Subject: Design of Machine Elements

Semester: 5th **Year:** 3rd

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Chapter-1

INTRODUCTION

1.1 Introduction to Machine Design & Classifications of Machine Design:

Introduction to Machine Design

The subject Machine Design is the creation of new and better machines and improving the existing ones. A newor better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived.

The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing amachine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Classifications of Machine Design:

The machine design may be classified as follows:

1. Adaptive design: In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. Development design: This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. Inthis case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. New design: This type of design needs lot of research, technical ability and creative thinking. Only thosedesigners who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows:

(a) Rational design: This type of design depends upon mathematical formulae of principle of mechanics.

(b) Empirical design: This type of design depends upon empirical formulae based on the practice and past experience.
(c) Industrial design: This type of design depends upon the production aspects to manufacture any machinecomponent in the industry.

(d) Optimum design: It is the best design for the given objective function under the specified constraints. It maybe achieved by minimising the undesirable effects.

(e) System design: It is the design of any complex mechanical system like a motor car.

(f) Element design: It is the design of any element of the mechanical system like piston, crankshaft, connectingrod, etc.

(g) Computer aided design. This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

1.2 Different mechanical engineering materials used in design with their uses and their mechanical and physical properties

Engineering Materials:

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.

1. Non-metals, such as glass, rubber, plastic, etc. The metals may be further classified as:

1. Ferrous metals.

The Ferrous metals are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

2. Non-ferrous metals.

The Non-ferrous metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost.

The following factors should be considered while selecting the material:

- Availability of the materials.
- Suitability of the materials for the working conditions in service.
- The cost of the materials.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals:

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include;

Strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness.

1. Strength: It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.
2. Stiffness: It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.
3. Elasticity: It is the property of a material to regain its original shape after deformation when the external forces are removed.
4. Plasticity: It is property of a material which retains the deformation produced under load permanently.
5. Ductility: It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice are mild steel, copper, aluminum, nickel, zinc, tin and lead.
6. Brittleness: It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Cast iron is a brittle material.
7. Malleability: It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. The malleable materials commonly used in engineering practice are lead, soft steel, wrought iron, copper and aluminum.
8. Toughness: It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated.
9. Resilience: It is the property of a material to absorb energy and to resist shock and impact loads. This property is essential for spring materials.
10. Creep: When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.
11. Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. This property is considered in designing shafts, connecting rods, springs, gears, etc.
12. Hardness: It is the property of the metals; it adopts many different properties such as resistance to wear, scratching, deformation and machinability etc. The hardness of a metal may be determined by the following tests:
    a) Brinell hardness test.

b) Rockwell hardness test.

c) Vickers hardness test.

1.3 Define working stress, yield stress, ultimate stress & factor of safety and stress –strain curve for M.S & C.I.

Working Stress:

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress.

yield Stress

The yield strength or yield stress is a material property and is the stress corresponding to the yield point at which the material begins to deform plastically.

Ultimate Stress

Ultimate tensile strength (UTS) is the maximum stress that a material can withstand before failure while being stretched or pulled.

Factor of Safety:

It is defined, in general, as the ratio of the maximum stress to the working stress.

Mathematically, **Factor of safety = Maximum stress / Working or design stress**

- In case of ductile materials; e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress.

In such cases;

Factor of safety = Yield point stress / Working or design stress

- In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

In such cases:

Factor of safety = Ultimate stress / Working or design stress

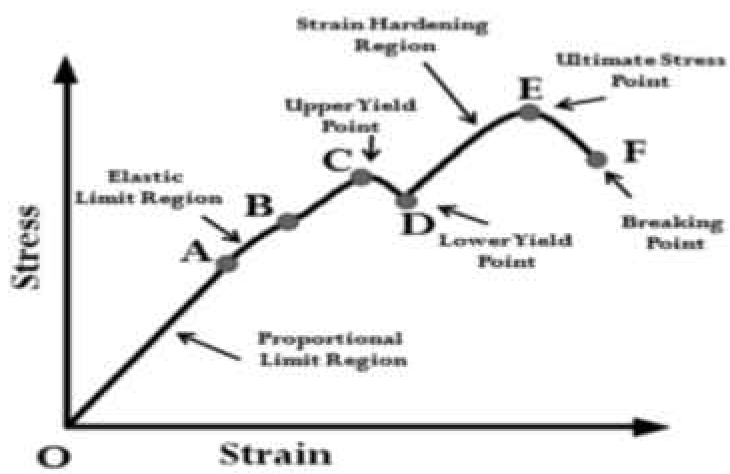
Stress Strain Curve for Mild Steel:

When a ductile material like mild steel is subjected to tensile force, it undergoes different stages before failure. Stress strain curve is the graphical representation of this stages. Different material may have different curve. Usually ductile materials follow similar pattern. so is for brittle materials. Here is the explanation

of stress strain curve for mild steel which is ductile material.

Here is the list of different stages when ductile material subjected to force till its failure.

- Proportional limit (point A)
- Elastic limit (point B)
- Yield point (upper yield point C and lower yield point D)
- Ultimate stress point (point E)
- Breaking point (point F)



Proportional limit:

As shown in stress strain curve for mild steel, up to the point A, stress and strain follow a relationship. This is known as Hook's law. Up to the limit of proportionality, stress directly followed the strain. This means ratio ofstress and strain remains constant

Elasticlimit:

Up to this limit (point B), is material will regain its original shape and size when unloaded. Point B is known as elastic point.

Yieldlimit:

When material is loaded beyond its elastic limit, it will not regain its original shape. There will be always some deformation.

Ultimatestress:

This is the maximum stress a material can bear. Value of stress corresponds to peak point on stress strain curvefor mild steel is the ultimate stress. It is denoted by point E in diagram.

Breakingstress:

Point on the stress strain curve where material fails, is known as breaking point. Stress correspond to this pointis known as breaking stress.

Stress Strain Curve for cast iron:

Materials which show very small elongation before they fracture are called brittle materials. The shape of curve for a Cast iron is shown in Fig.(b) and is typical of many brittle materials such as Carbon steel, concrete and high strength light alloys. For most brittle materials the permanent elongation (i.e., increase in length) is less than 5%.

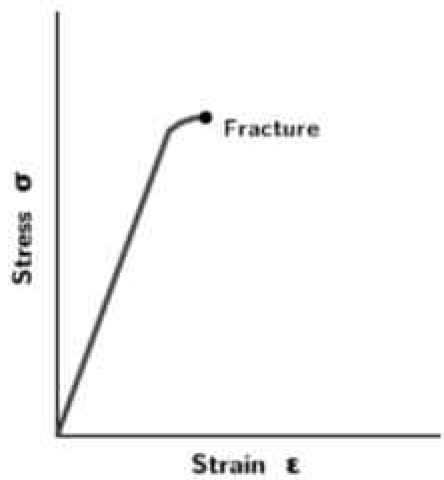


Fig.b stress-strain curve for cast iron.

The ultimate strength is coincident with the fracture point. In this case, no necking occurs.

1.4 Modes of Failure (By elastic deflection, general yielding & fracture)

Modes of failure:

A mechanical component may fail i.e. it may be unable to perform its function satisfactorily, as a result of anyone of the following three modes of failure:

1. Failure by elastic deflection.
2. Failure by Yielding
3. failure by Fracture

1. Failure by elastic deflection

In applications like transmission shaft supporting the gears, the maximum force acting on the shaft, without effecting it performance is limited by the permissible elastic deflection. Sometimes the elastic deflection results in unstable conditions, such as buckling of columns or vibrations. The design of mechanical component, in all these cases, is based on the permissible lateral or torsional deflection. The stresses induced in the component are not significant and properties of the material are not of primary importance. The moduli of elasticity and rigidity are the important properties and dimension of component are determined by the load deflection equation.

In short, in a components like : columns, beams, shafts etc.,the torsional deflection in an elastic region is termed as failure of the component

2. Failure by Yielding

For ductile material deformation occurs after the yield point, resulting in permanent deformation of the machine element which ultimately breaks at breaking point. Hence for ductile materials, failure is usually considered to have occurred when yielding i.e. plastic deformation reach a limit, when engineering usefulness of the part is destroyed, even through there is no rupture or fracture of machine part. Thus, the yield point is criterion of failure of ductile materials subjected to static loading.

In short, when a mechanical component, made of ductile material, undergoes yielding or plastic deformation, its functional utility comes to an end and it is termed as failure of the component. Such failure is known as **Plastic failure.**

3. Failure by fracture

In case of brittle materials the yield point and ultimate strain is very nearly equal to unity. So brittlematerials are considered to have failed by fracture with little or no permanent deformation.

Sudden separation or a breakage of a material along the cross-section normal to the direction of stress is known as fracture. Fracture is a sudden failure without plastic deformation. The failure of components made of brittle material is due to fracture.

1.5 Factors governing the design of machine elements:

1. Type of load and stresses caused by the load.

The load, on a machine component, may act in several ways due to which the internal stresses are set up.

2. Motion of the parts or kinematics of the machine.

The successful operation of any ma-chine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be :

(a) Rectilinear motion which includes unidirectional and reciprocating motions.

(b) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(c) Constant velocity.

(d) Constant or variable acceleration.

3. Selection of materials.

It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are : strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc.

4. Form and size of the parts.

The form and size are based on judgement. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. Frictional resistance and lubrication

There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. Convenient and economical features.

In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling .The economical operation of a machine which is to be used for production, or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. Use of standard parts.

The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. Safety of operation.

Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. Workshop facilities.

A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods forcasting, handling and machining special parts.

10. Number of machines to be manufactured.

The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue Design considerations play important role in the successful production of machines. Expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. Cost of construction.

The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. Assembling.

Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

1.6 General Procedure in Machine Design:

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways.However, the general procedure to solve a design problem is as follows:

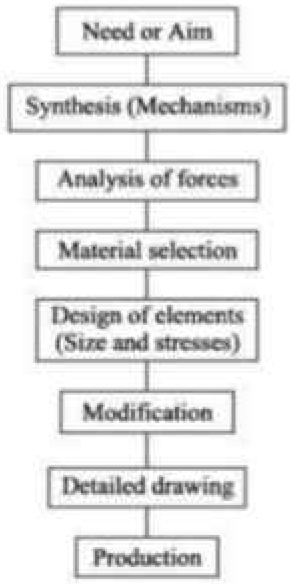


Fig. General Machine Design Procedure

1. Recognition of need: First of all, make a complete statement of the problem, indicating the need, aim orpurpose for which the machine is to be designed.

2. Synthesis (Mechanisms): Select the will give the desired motion.

3. Analysis of forces: Find the forces acting on each member of the machine and the energy transmitted byeach member.

4. Material selection: Select the material best suited for each member of the machine.

5. Design of elements (Size and Stresses): considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform thanthe permissible limit.

6. Modification: Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. Detailed drawing: Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. Production. The component, as per the drawing, is manufactured in the workshop. The flow chart for thegeneral procedure in machine design is shown in Fig.

Chapter-2

DESIGN OF FASTENING ELEMENTS

2.1 Joints and their classification.

Fasteners: It is a Mechanical Joints which is used to become a fixed / attaches to something or holds something in place.
The Fastenings may be classified into the following two groups:

1. The Permanent Fastenings are those fastenings which cannot be disassembled withoutdestroying the connecting components. Examples: Welded joint, Rivet joint.
2. The Temporary or Detachable Fastenings are those fastenings which can be disassembledwithout destroying the connecting components.
   Examples: 1.Thread Joints
   a. Bolted Joints
   b. Screws Joints
   2. Keys
   3. Coupling
   4. Pins Joints
   a. Cotters Joints
   b. Knuckle Joints
   5. Pipe Joints

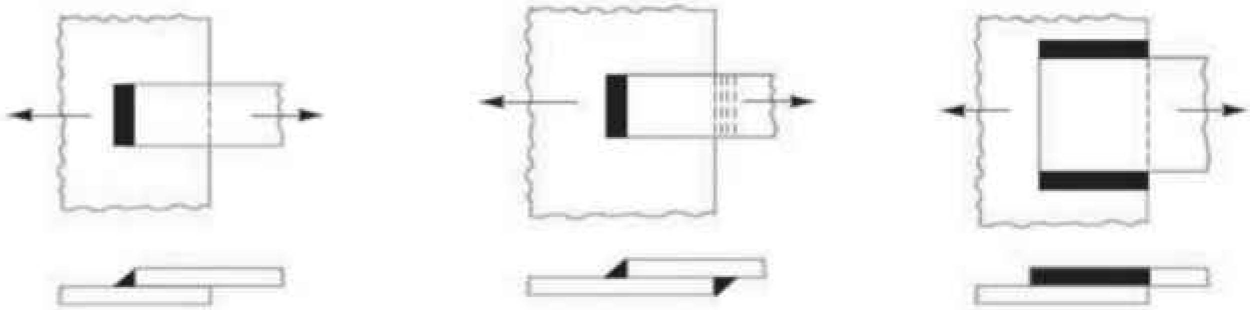
2.2 Types of welded joints .

Welded joint:
Welding can be defined as a process of joining metallic parts by heating to a suitable temperature with orwithout the application of pressure.
Welding is an economical and efficient method for obtaining a permanent joint of metallic parts.Two distinct application of welding

1. Can be used as a substitute for a riveted joint
2. Welded structure as an alternative method for casting or forging.

Types of welded joints
Following two types of welded joints
1. Lap joint or fillet joint,
2. Butt joint.

Lap Joint
The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular.
The fillet joints may be
1. Single transverse fillet, 2. Double transverse fillet and 3. Parallel fillet joints.
A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

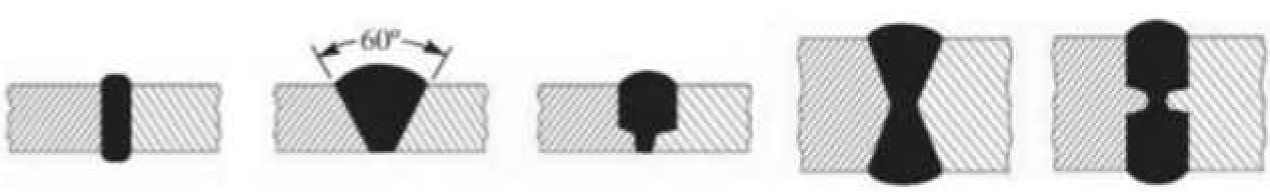


(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. below. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.

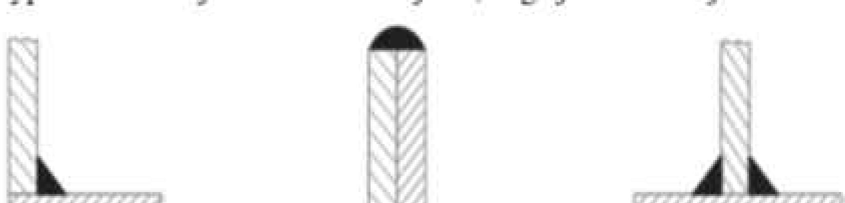
The butt joints may be
1. Square butt joint
2. Single V-butt joint
3. Single U-butt joint,
4. Double V-butt joint,
5. Double U-butt joint.



(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

Other Joints

The other type of welded joints are corner joint, edge joint and T-joint as shown in Fig below



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

The main considerations involved in the selection of weld type are:
1. The shape of the welded component required,
2. The thickness of the plates to be welded, and
3. The direction of the forces applied.

2.3 Welding advantages over riveting:

Following are the advantages and disadvantages of welded joints over riveted joints.
Advantages

1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (i.e. circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

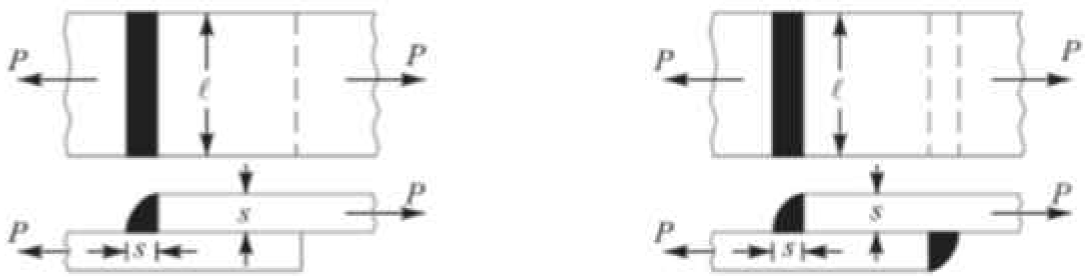
Disadvantages

1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
2. It requires a highly skilled labour and supervision.
3. Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

2.4 Design of welded joints

Strength of Transverse Fillet Welded Joints

We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. (a) and (b) respectively



(*a*) Single transverse fillet weld. (*b*) Double transverse fillet weld.

Transverse fillet welds.

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC. The enlarged view of the fillet is shown in Fig. The length of each side is known as leg or size of the weld and the perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as throat thickness. The minimum area of the weld is obtained at the throat BD, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (*BD*),

s = Leg or size of weld,

= Thickness of plate, and

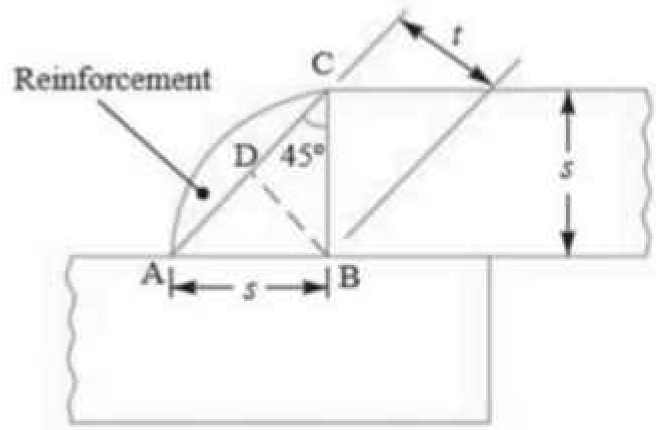
l = Length of weld,

From Fig. we find that the throat thickness,

$$t = s \times \sin 45° = 0.707\, s$$

∴ *Minimum area of the weld or throat area,

$$A = \text{Throat thickness} \times \text{Length of weld} = t \times l = 0.707\, s \times l$$



Enlarged view of a fillet weld.

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

$$P = \text{Throat area} \times \text{Allowable tensile stress} = 0.707\, s \times l \times \sigma_t$$

and tensile strength of the joint for double fillet weld,

$$P = 2 \times 0.707\, s \times l \times \sigma_t = 1.414\, s \times l \times \sigma_t$$

Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig.(a). We have already discussed in the previous article, that the minimum area of weld or the throat area

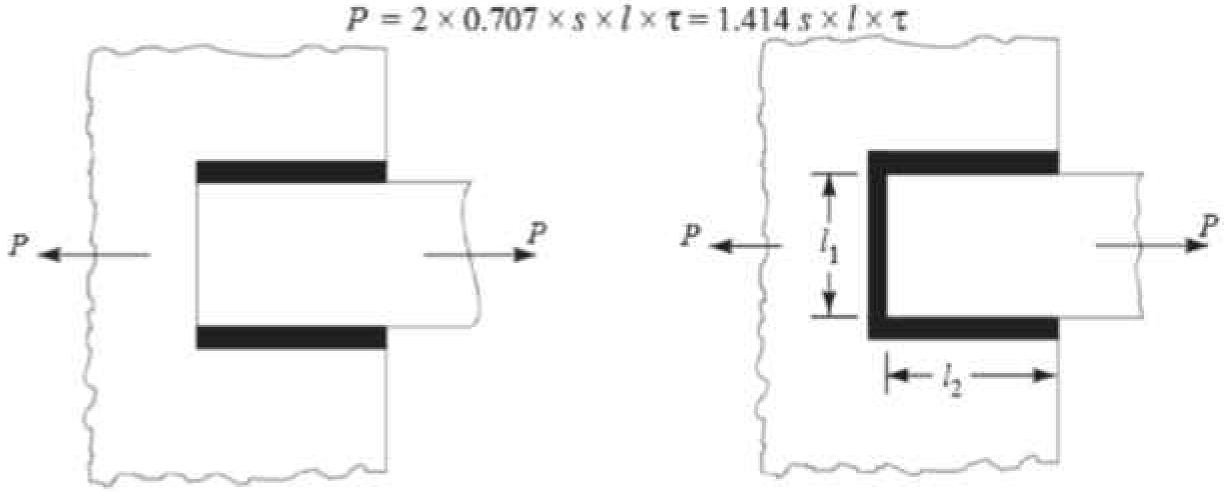
$$A = 0.707\ s \times l$$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

$$P = \text{Throat area} \times \text{Allowable shear stress} = 0.707\ s \times l \times \tau$$

and shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\ s \times l \times \tau$$



(*a*) Double parallel fillet weld.

(*b*) Combination of transverse and parallel fillet weld.

Notes: 1. If there is a combination of single transverse and double parallel fillet welds as shown in Fig. (*b*), then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds. Mathematically,

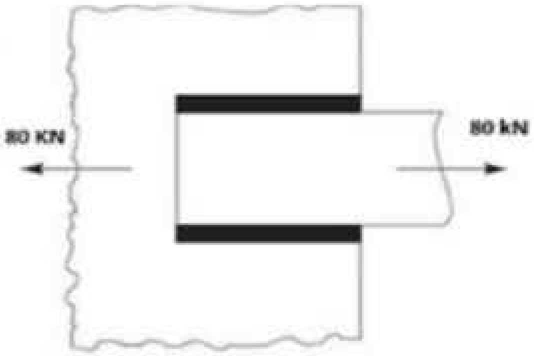
$$P = 0.707s \times l_1 \times \sigma_t + 1.414\ s \times l_2 \times \tau$$

where l_1 is normally the width of the plate.

2. In order to allow for starting and stopping of the bead, 12.5 mm should be added to the length of each weld obtained by the above expression.

3. For reinforced fillet welds, the throat dimension may be taken as 0.85 *t*.

Example 1 *A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.*



(a) Double parallel fillet weld.

Solution. Given: *Width = 100 mm ; Thickness = 10 mm ; P = 80 kN = 80 × 10^3 N ; τ = 55 MPa = 55 N/mm^2

Let l = Length of weld, and

s = Size of weld = Plate thickness = 10 mm ... (Given)

We know that maximum load which the plates can carry for double parallel fillet weld (P),

$$80 \times 10^3 = 1.414 \times s \times l \times \tau = 1.414 \times 10 \times l \times 55 = 778\, l$$

$$\therefore \quad l = 80 \times 10^3 / 778 = 103 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 103 + 12.5 = 115.5 \text{ mm} \quad \textbf{Ans.}$$

Example 2

A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 MPa. Consider the joint first under static loading and then under fatigue loading.

Solution. Given: *Width = 100 mm ; Thickness = 12.5 mm ; P = 50 kN = 50 × 10^3N ; τ = 56 MPa = 56 N/mm^2

Length of weld for static loading

Let l = Length of weld, and

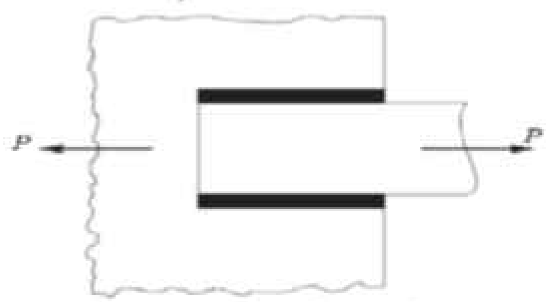
s = Size of weld = Plate thickness = 12.5 mm ... (Given)

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414\, s \times l \times \tau$$
$$= 1.414 \times 12.5 \times l \times 56 = 990\, l$$
$$\therefore \quad l = 50 \times 10^3 / 990 = 50.5 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 50.5 + 12.5 = 63 \text{ mm} \quad \textbf{Ans.}$$



Length of weld for fatigue loading

From Table 10.6, we find that the stress concentration factor for parallel fillet welding is 2.7.

$\therefore$ Permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

We know that the maximum load which the plate can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414\, s \times l \times \tau = 1.414 \times 12.5 \times l \times 20.74 = 367\, l$$

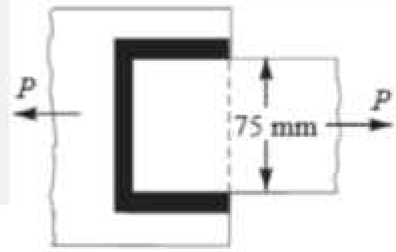
$$\therefore \quad l = 50 \times 10^3 / 367 = 136.2 \text{ mm}$$

Adding 12.5 for starting and stopping of weld run, we have

$$l = 136.2 + 12.5 = 148.7 \text{ mm} \quad \textbf{Ans.}$$

Example 3 *A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. 10.15. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively.*

Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.



Solution. Given : Width = 75 mm ; Thickness = 12.5 mm ; σ_t = 70 MPa = 70 N/mm^2 ; τ = 56 MPa = 56 N/mm^2.

The effective length of weld (l_1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$\therefore \quad l_1 = 75 - 12.5 = 62.5$ mm

Length of each parallel fillet for static loading

Let l_2 = Length of each parallel fillet.

We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} = 75 \times 12.5 \times 70 = 65\,625 \text{ N}$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70 = 38\,664 \text{ N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5 \times l_2 \times 56 = 990\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65\,625 = P_1 + P_2 = 38\,664 + 990\, l_2 \quad \text{or} \quad l_2 = 27.2 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 27.2 + 12.5 = 39.7 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

Length of each parallel fillet for fatigue loading

From Table 10.6, we find that the stress concentration factor for transverse welds is 1.5 and for parallel fillet welds is 2.7.

$\therefore$ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm}^2$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7 = 25\,795 \text{ N}$$

and load carried by double parallel fillet weld,

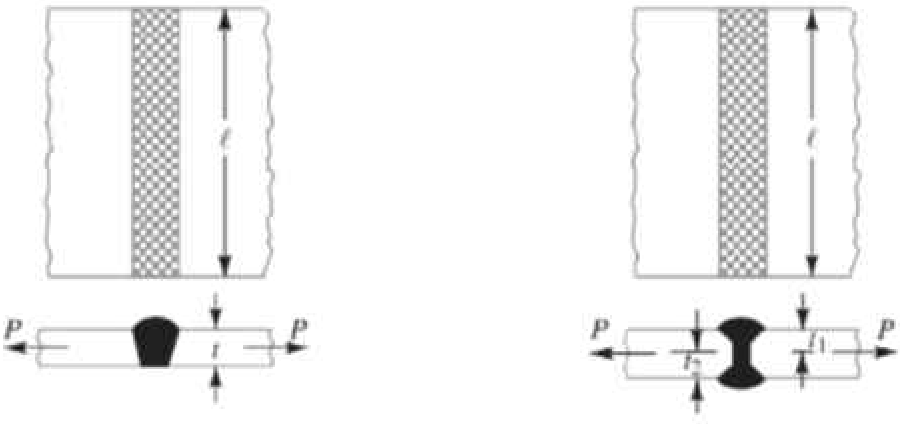
$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5\, l_2 \times 20.74 = 366\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65\,625 = P_1 + P_2 = 25\,795 + 366\, l_2 \quad \text{or} \quad l_2 = 108.8 \text{ mm}$$

Strength of Butt Joints

The butt joints are designed for tension or compression. Consider a single V-butt joint as shown in Fig. (*a*).



(*a*) Single *V*-butt joint. (*b*) Double *V*-butt joint.

In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is equal to thickness of plates.

∴ Tensile strength of the butt joint (single-*V* or square butt joint),

$$P = t \times l \times \sigma_t$$

where l = Length of weld. It is generally equal to the width of plate.

and tensile strength for double-*V* butt joint as shown in Fig. (*b*) is given by

$$P = (t_1 + t_2)\, l \times \sigma_t$$

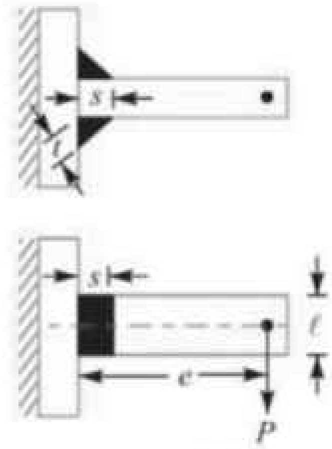
where t_1 = Throat thickness at the top, and

t_2 = Throat thickness at the bottom.

It may be noted that size of the weld should be greater than the thickness of the plate, but it may be less. The following table shows recommended minimum size of the welds.

Eccentrically Loaded Welded Joints

An eccentric load may be imposed on welded joints in many ways. The stresses induced on the joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses. When the shear and bending stresses are simultaneously present in a joint (see case 1), then maximum stresses are as follows:



Maximum normal stress,

$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

where σ_b = Bending stress, and

τ = Shear stress.

When the stresses are of the same nature, these may be combined vectorially (see case 2).

We shall now discuss the two cases of eccentric loading as follows:

Case 1

Consider a T-joint fixed at one end and subjected to an eccentric load P at a distance e as shown in Fig. 10.22.

Let s = Size of weld,

l = Length of weld, and

t = Throat thickness.

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force P acting at the welds, and
2. Bending stress due to the bending moment $P \times e$.

We know that area at the throat,

A = Throat thickness × Length of weld

$= t \times l \times 2 = 2\,t \times l$ ... (For double fillet weld)

$= 2 \times 0.707\,s \times l = 1.414\,s \times l$ ... ($\because t = s \cos 45° = 0.707\,s$)

$\therefore$ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\, s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \qquad \text{...(For both sides weld)}$$

$$= \frac{0.707\, s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

Bending moment, $M = P \times e$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\, P \times e}{s \times l^2}$

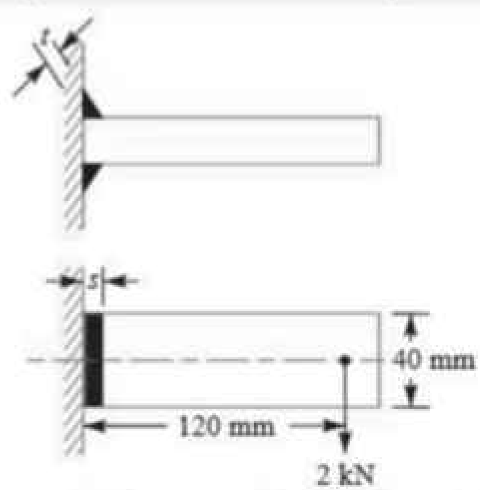
We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2}$$

Example 4 *A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.*



Solution. Given: P = 2kN = 2000 N ; e = 120 mm ; l = 40 mm ; τ_{max} = 25 MPa = 25 N/mm^2

Let s = Size of weld in mm, and

t = Throat thickness.

The joint, as shown in Fig. 10.24, will be subjected to direct shear stress due to the shear force, P = 2000 N and bending stress due to the bending moment of $P \times e$.

We know that area at the throat,

$$A = 2t \times l = 2 \times 0.707\, s \times l$$
$$= 1.414\, s \times l$$
$$= 1.414\, s \times 40 = 56.56 \times s \text{ mm}^2$$

$\therefore$ Shear stress, $\tau = \dfrac{P}{A} = \dfrac{2000}{56.56 \times s} = \dfrac{35.4}{s}$ N/mm^2

Bending moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3$ N-mm

Section modulus of the weld through the throat,

$$Z = \frac{s \times l^2}{4.242} = \frac{s\,(40)^2}{4.242} = 377 \times s \text{ mm}^3$$

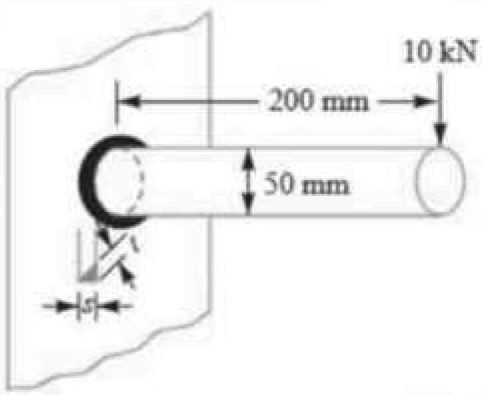
$\therefore$ Bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{240 \times 10^3}{377 \times s} = \dfrac{636.6}{s}$ N/mm^2

We know that maximum shear stress (τ_{max}),

$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore$ $s = 320.3 / 25 = 12.8$ mm **Ans.**

Example 5 *A 50 mm diameter solid shaft is welded to a flat plate as shown in Fig. If the size of the weld is 15 mm, find the maximum normal and shear stress in the weld.*



Solution. Given : $D = 50$ mm ; $s = 15$ mm ; $P = 10$ kN $= 10\,000$ N ; $e = 200$ mm

Let t = Throat thickness.

The joint, as shown in Fig. 10.25, is subjected to direct shear stress and the bending stress. We know that the throat area for a circular fillet weld,

$$A = t \times \pi D = 0.707\, s \times \pi D$$
$$= 0.707 \times 15 \times \pi \times 50$$
$$= 1666 \text{ mm}^2$$

$\therefore$ Direct shear stress,

$$\tau = \frac{P}{A} = \frac{10\,000}{1666} = 6 \text{ N/mm}^2 = 6 \text{ MPa}$$

We know that bending moment,

$$M = P \times e = 10\,000 \times 200 = 2 \times 10^6 \text{ N-mm}$$

From Table 10.7, we find that for a circular section, section modulus,

$$Z = \frac{\pi\, t\, D^2}{4} = \frac{\pi \times 0.707\, s \times D^2}{4} = \frac{\pi \times 0.707 \times 15\,(50)^2}{4} = 20\,825 \text{ mm}^3$$

$\therefore$ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{2 \times 10^6}{20\,825} = 96 \text{ N/mm}^2 = 96 \text{MPa}$$

Maximum shear stress

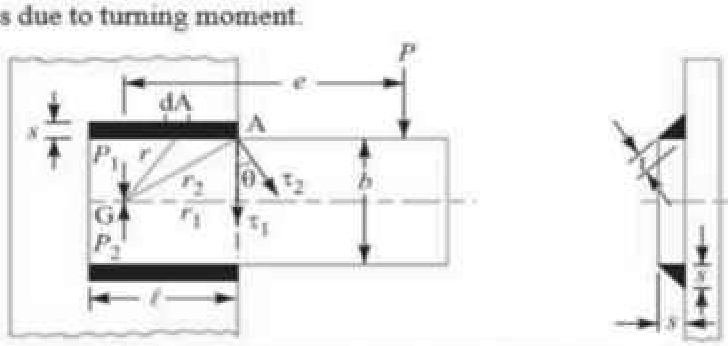
We know that the maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(96)^2 + 4\times 6^2} = 48.4 \text{ MPa} \quad \text{Ans.}$$

Case 2

When a welded joint is loaded eccentrically as shown in Fig. the following two types of the stresses are induced:

1. Direct or primary shear stress, and
2. Shear stress due to turning moment.



Eccentrically loaded welded joint.

Let P = Eccentric load,

e = Eccentricity *i.e.* perpendicular distance between the line of action of load and centre of gravity (G) of the throat section or fillets,

l = Length of single weld,

s = Size or leg of weld, and

t = Throat thickness.

Let two loads P_1 and P_2 (each equal to P) are introduced at the centre of gravity 'G' of the weld system. The effect of load $P_1 = P$ is to produce direct shear stress which is assumed to be uniform over the entire weld length. The effect of load $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends of rotate the joint about the centre of gravity 'G' of the weld system. Due to the turning moment, secondary shear stress is induced.

We know that the direct or primary shear stress,

$$\tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{2\,t \times l}$$

$$= \frac{P}{2 \times 0.707\, s \times l} = \frac{P}{1.414\, s \times l}$$

... ($\because$ Throat area for single fillet weld $= t \times l = 0.707\, s \times l$)

Since the shear stress produced due to the turning moment ($T = P \times e$) at any section is proportional to its radial distance from G, therefore stress due to $P \times e$ at the point A is proportional to AG (r_2) and is in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = \text{Constant}$$

or

$$\tau = \frac{\tau_2}{r_2} \times r \qquad ...(i)$$

where τ_2 is the shear stress at the maximum distance (r_2) and τ is the shear stress at any distance r.

Consider a small section of the weld having area dA at a distance r from G.

∴ Shear force on this small section

$$= \tau \times dA$$

and turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2 \qquad \text{... [From equation (i)]}$$

∴ Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times dA \times r^2 = \frac{\tau_2}{r_2} \int dA \times r^2$$

$$= \frac{\tau_2}{r_2} \times J \qquad \left(\because J = \int dA \times r^2\right)$$

where J = Polar moment of inertia of the throat area about G.

∴ Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

∴ Resultant shear stress at A,

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

where θ = Angle between τ_1 and τ_2, and

$$\cos\theta = r_1 / r_2$$

Note: The polar moment of inertia of the throat area (A) about the centre of gravity (G) is obtained by the parallel axis theorem, *i.e.*

$$J = 2\,[I_{xx} + A \times x^2] \qquad \text{... (}\because \text{ of double fillet weld)}$$

$$= 2\left[\frac{A \times l^2}{12} + A \times x^2\right] = 2A\left(\frac{l_2}{12} + x^2\right)$$

where A = Throat area = $t \times l = 0.707\, s \times l$,

l = Length of weld, and

x = Perpendicular distance between the two parallel axes.

Example 6 . *A bracket carrying a load of 15 kN is to be welded as shown in Fig. Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.*

Solution. Given : $P = 15$ kN $= 15 \times 10^3$ N ; $\tau = 80$ MPa $= 80$ N/mm^2 ; $b = 80$ mm ; $l = 50$ mm; $e = 125$ mm

Let s = Size of weld in mm, and

t = Throat thickness.

We know that the throat area,

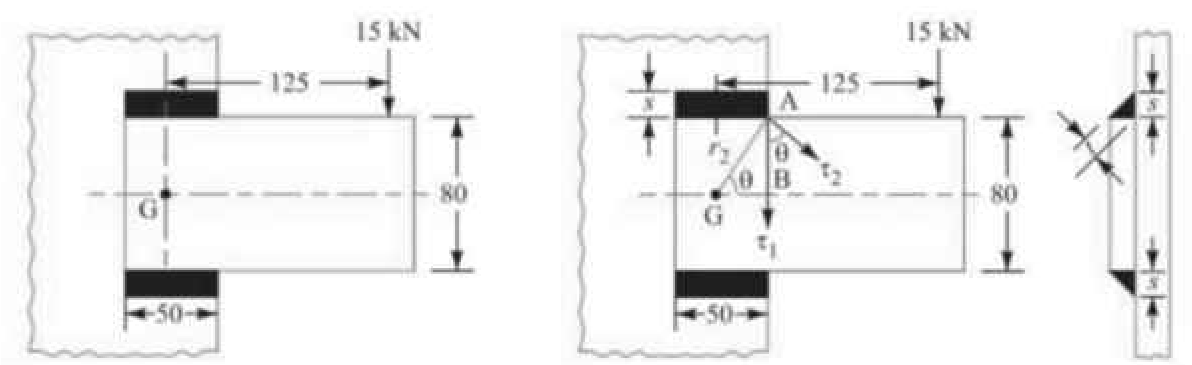
$$A = 2 \times t \times l = 2 \times 0.707\, s \times l$$
$$= 1.414\, s \times l = 1.414 \times s \times 50 = 70.7\, s \text{ mm}^2$$

∴ Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{15 \times 10^3}{70.7\, s} = \frac{212}{s} \text{ N/mm}^2$$

From Table 10.7, we find that for such a section, the polar moment of inertia of the throat area of the weld about G is

$$J = \frac{t\,l\,(3b^2 + l^2)}{6} = \frac{0.707\, s \times 50\,[3\,(80)^2 + (50)^2]}{6} \text{ mm}^4$$
$$= 127\,850\, s \text{ mm}^4 \qquad \text{... } (\because t = 0.707\, s)$$



All dimensions in mm.

From Fig. we find that $AB = 40$ mm and $BG = r_1 = 25$ mm.

∴ Maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(40)^2 + (25)^2} = 47 \text{ mm}$$

Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{15 \times 10^3 \times 125 \times 47}{127\ 850\ s} = \frac{689.3}{s} \text{ N/mm}^2$$

and

$$\cos\theta = \frac{r_1}{r_2} = \frac{25}{47} = 0.532$$

We know that resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2 \cos\theta}$$

$$80 = \sqrt{\left(\frac{212}{s}\right)^2 + \left(\frac{689.3}{s}\right)^2 + 2 \times \frac{212}{s} \times \frac{689.3}{s} \times 0.532} = \frac{822}{s}$$

$\therefore$ $s = 822 / 80 = 10.3$ mm **Ans.**

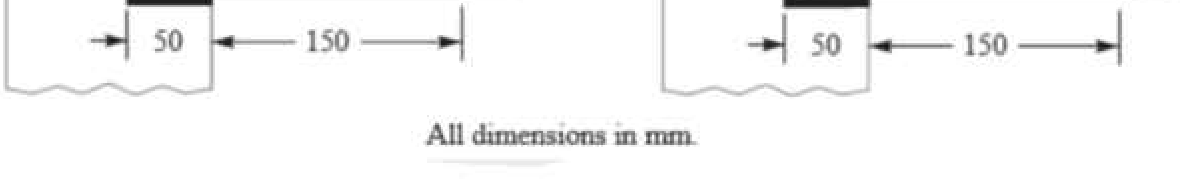
Example 7 . *A rectangular steel plate is welded as a cantilever to a vertical column and supports a single concentrated load P, as shown in Fig. 10.30.*

Determine the weld size if shear stress in the same is not to exceed 140 MPa.

Solution. Given : $P = 60$ kN $= 60 \times 10^3$ N ; $b = 100$ mm ; $l = 50$ mm ; $\tau = 140$ MPa $= 140$ N/mm^2

Let s = Weld size, and

t = Throat thickness.



All dimensions in mm.

First of all, let us find the centre of gravity (G) of the weld system, as shown in Fig.

Let x be the distance of centre of gravity (G) from the left hand edge of the weld system. From Table 10.7, we find that for a section as shown in Fig. 10.31,

$$x = \frac{l^2}{2l + b} = \frac{(50)^2}{2 \times 50 + 100} = 12.5 \text{ mm}$$

and polar moment of inertia of the throat area of the weld system about G,

$$J = t\left[\frac{(b + 2l)^3}{12} - \frac{l^2\,(b + l)^2}{b + 2l}\right]$$

$$= 0.707\,s\left[\frac{(100 + 2 \times 50)^3}{12} - \frac{(50)^2\,(100 + 50)^2}{100 + 2 \times 50}\right] \ldots (\because t = 0.707\,s)$$

$$= 0.707s\,[670 \times 10^3 - 281 \times 10^3] = 275 \times 10^3\,s \text{ mm}^4$$

Distance of load from the centre of gravity (G) *i.e.* eccentricity,

$$e = 150 + 50 - 12.5 = 187.5 \text{ mm}$$

$$r_1 = BG = 50 - x = 50 - 12.5 = 37.5 \text{ mm}$$

$$AB = 100 / 2 = 50 \text{ mm}$$

We know that maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(50)^2 + (37.5)^2} = 62.5 \text{ mm}$$

$$\therefore \qquad \cos\theta = \frac{r_1}{r_2} = \frac{37.5}{62.5} = 0.6$$

We know that throat area of the weld system,

$$A = 2 \times 0.707s \times l + 0.707s \times b = 0.707\,s\,(2l + b)$$

$$= 0.707s\,(2 \times 50 + 100) = 141.4\,s \text{ mm}^2$$

$\therefore$ Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{60 \times 10^3}{141.4s} = \frac{424}{s} \text{ N/mm}^2$$

and shear stress due to the turning moment or secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{60 \times 10^3 \times 187.5 \times 62.5}{275 \times 10^3\,s} = \frac{2557}{s} \text{ N/mm}^2$$

We know that the resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2 \times \cos\theta}$$

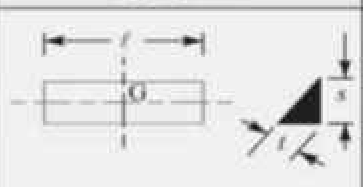
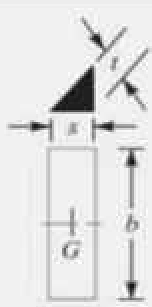
$$140 = \sqrt{\left(\frac{424}{s}\right)^2 + \left(\frac{2557}{s}\right)^2 + 2 \times \frac{424}{s} \times \frac{2557}{s} \times 0.6} = \frac{2832}{s}$$

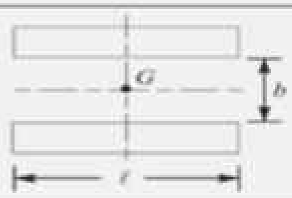
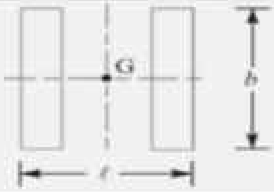
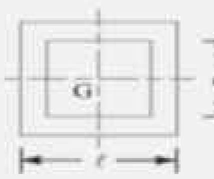
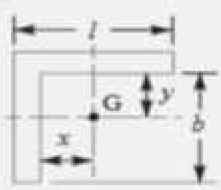
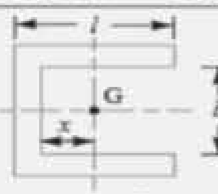
$\therefore$ $$s = 2832 / 140 = 20.23 \text{ mm} \quad \textbf{Ans.}$$

Polar Moment of Inertia and Section Modulus of Welds

The following table shows the values of polar moment of inertia of the throat area about the centre of gravity '*G*' and section modulus for some important types of welds which may be used for eccentric loading.

Table Polar moment of inertia and section modulus of welds.

S.No	*Type of weld*	*Polar moment of inertia (J)*	*Section modulus (Z)*
1.		$\frac{t.l^3}{12}$	—
2.		$\frac{t.b^3}{12}$	$\frac{t.b^2}{6}$

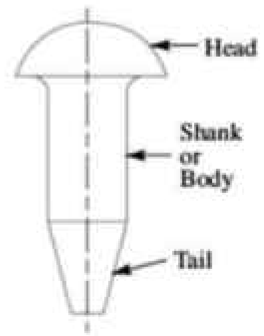
S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
3.		$\frac{t.l(3b^2 + l^2)}{6}$	$t.b.l$
4.		$\frac{t.b(b^2 + 3l^2)}{6}$	$\frac{t.b^2}{3}$
5.		$\frac{t(b + l)^3}{6}$	$t\left(b.l + \frac{b^2}{3}\right)$
6.	 $x = \frac{l^2}{2(l+b)}, y = \frac{b^2}{2(l+b)}$	$t\left[\frac{(b+l)^4 - 6b^2l^2}{12(l+b)}\right]$	$t\left(\frac{4l.b + b^2}{6}\right)$ (Top) $t\left[\frac{b^2(4lb + b)}{6(2l + b)}\right]$ (Bottom)
7.	 $x = \frac{l^2}{2l + b}$	$t\left[\frac{(b+2l)^3}{12} - \frac{l^2(b+l)^2}{b+2l}\right]$	$t\left(l.b + \frac{b^2}{6}\right)$
8.		$\frac{\pi t d^3}{4}$	$\frac{\pi t d^2}{4}$

2.5 State types of riveted joints and types of rivets.

Riveted joint:

The rivets are used to make permanent fastening between the two or more plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.

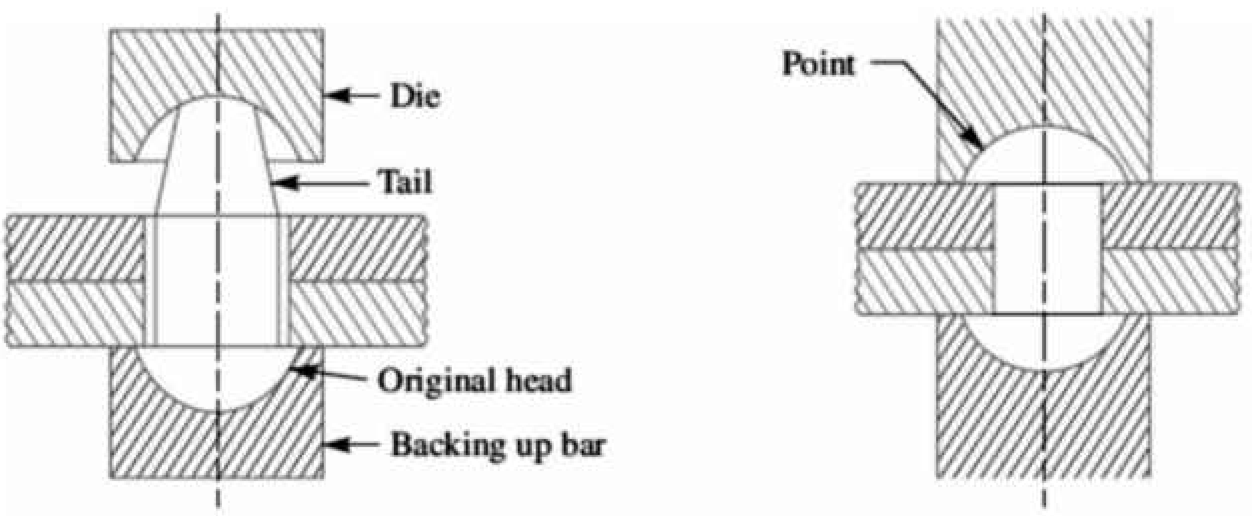
A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called shank or body and lowerportion of shank is known as tail.



Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and toprevent leakage as in a boiler or in a ship hull (The frame or body of ship).

When two plates are to be fastened together by a rivet as shows below, the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work.



(*a*) **Initial position.** (*b*) **Final position.**

In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.
A cold rivet or a red hot rivet is introduced into the plates and the point (i.e. second head) is then formed.
When a cold rivet is used, the process is known as cold riveting and when a hot rivet is used, the process is

known as hot riveting.

The cold riveting process is used for structural joints while hot riveting is used to make leak proof joints.

Notes: 1. For steel rivets upto 12 mm diameter, the cold riveting process may be used while for larger diameter Rivets, hot riveting process are used.

2. In case of long rivets, only the tail is heated and not the whole shank

Material of Rivets:

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminum or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used.

Types of Rivets:

1. Button Head
2. Counter sunk Head
3. Oval counter Head
4. Pan Head
5. Conical Head

Types of Riveted Joints

1. According to purpose
2. According to position of plates connected
3. According to arrangement of rivets

1. **According to purpose**:

   a) Strong Joints: In these Joints strength is the only criterion. Eg: Beams, Trusses and Machine Joints.

   b) Tight joints: These joints provide strength as well as are leak proofagainst low pressure. Eg: Reservoir, Containers and tanks.

   c) Strong-Tight Joints: These are the joints applied in boilers and pressure Vessels and ensure both strength and leak proofness.

2. **According to position of plates**:

   - **Lap Joint:** A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.
   - **Butt Joint:** A butt joint is that in which the main plates are touching each other and a cover plate (i.e. Strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types:

     a. In a single strap butt joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

     b. In a double strap butt joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

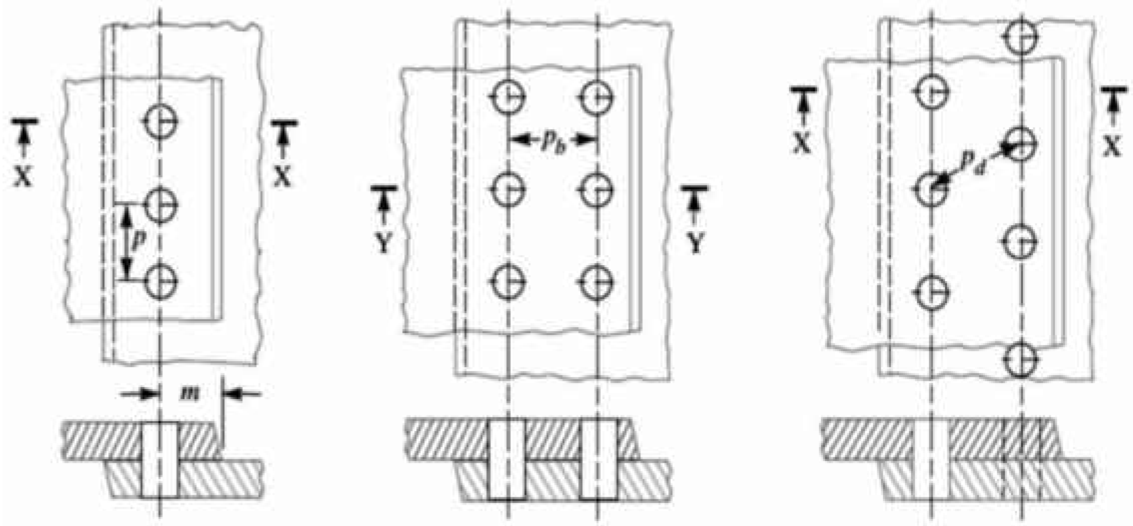
3. **<u>According to arrangement of rivets</u>:**

a. A **single riveted joint** is that in which there is a single row of rivets in a lap joint as shown in Fig. and there is a single row of rivets on each side in a butt joint.

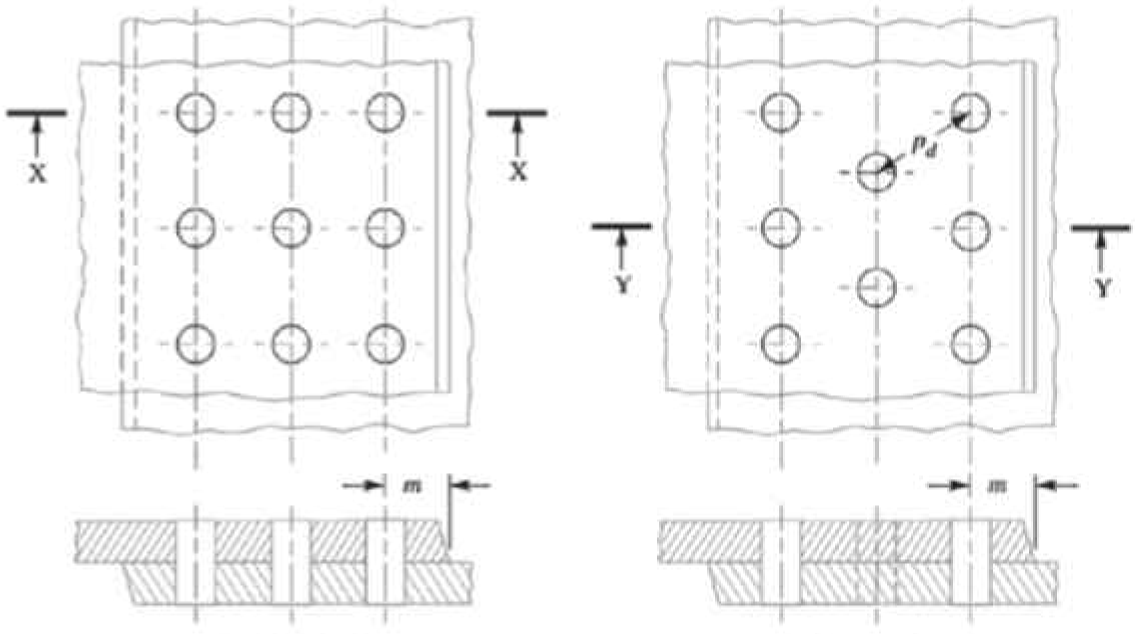
b. A **double riveted joint** is that in which there are two rows of rivets in a lap joint as shown in Fig. and there are two rows of rivets on each side in a butt joint.

Notes : 1. When the rivets in the various rows are opposite to each other, as shown in Fig, then the joint is said to be chain riveted.

if the rivets in the adjacent rows are staggered in such a way that every rivet is in the middle of the two rivets of the opposite row as shown then the joint is said to be zig-zag riveted.

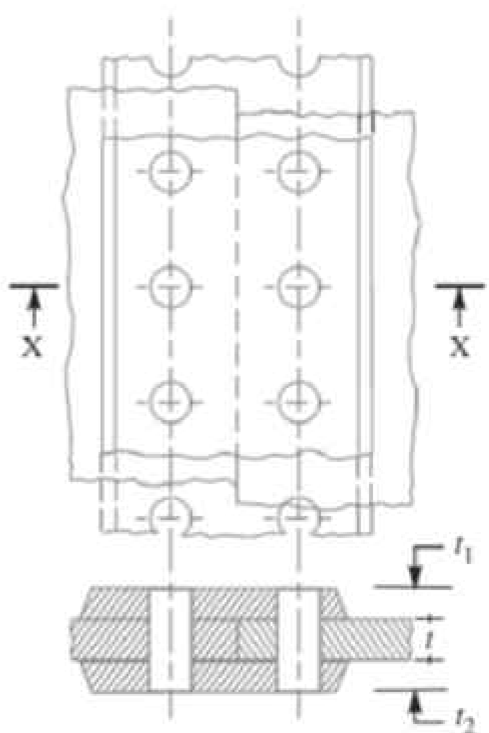


(*a*) Single riveted lap joint. (*b*) Double riveted lap joint (Chain riveting). (*c*) Double riveted lap joint (Zig-zag riveting).

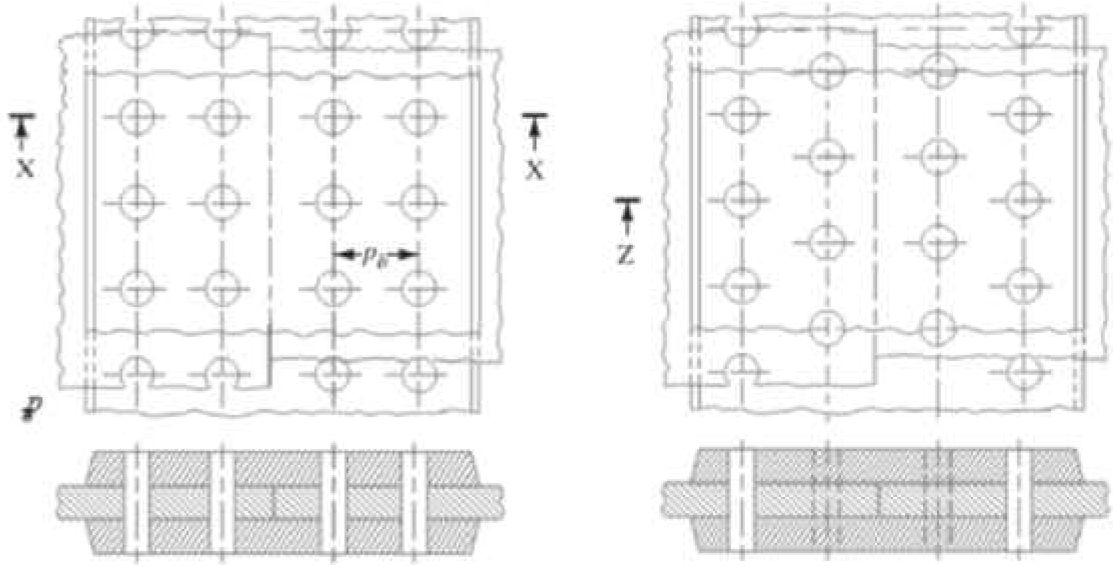


(a) Chain riveting. (b) Zig-zag riveting.

Triple riveted lap joint.

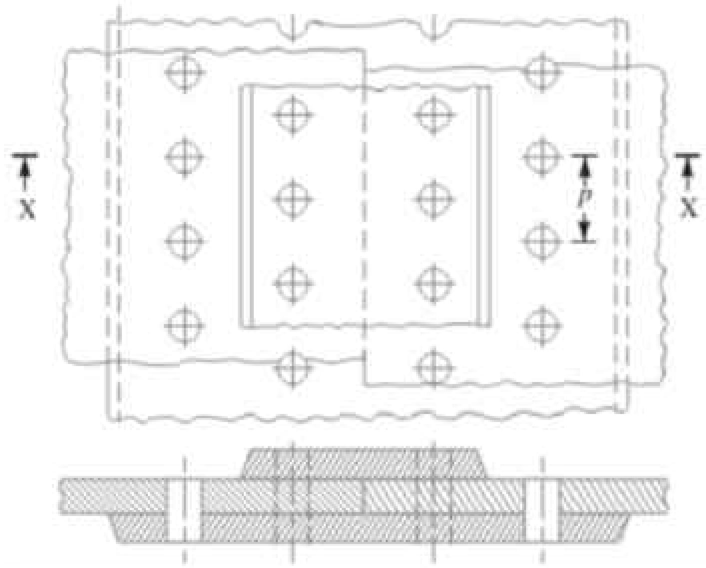


Single riveted double strap butt joint.



(*a*) Chain riveting. (*b*) Zig-zag riveting.

Double riveted double strap (equal) butt joints.

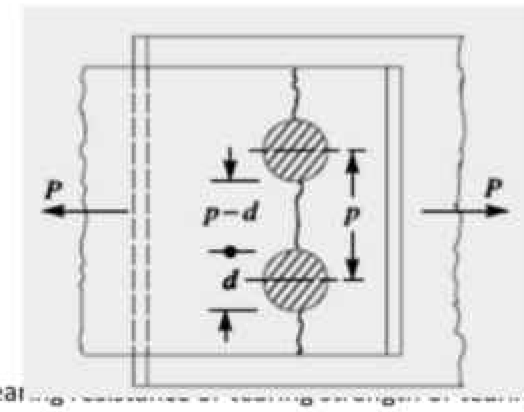


. Double riveted double strap (unequal) butt joint with zig-zag riveting.

<u>Important terms of Riveted joints</u>:

1. **Pitch (p)**: It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig(a) It is usually denoted by p
2. **Back pitch (P_b)**: The Distance between two adjacent rows of rivets.
3. **Diagonal pitch(P_d)**: It is the distance between the centers of the rivets in adjacent rows of zigzag Riveted joint as shown in Fig. (c) It is usually denoted by p_d.
4. **Margin (m)**: It is the distance between center of a rivet hole and nearest edge of the plate.

2.6 Modes of Failures of a Riveted Joint



1. **Tearing of the plate at the section weakened by holes:** Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate.

Let, p = Pitch of the rivets,
d = Diameter of the rivet hole,
t = Thickness of the plate, and
σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

$$A_t = (p - d)\,t$$

Tearing resistance or pull required to tear off the plate per pitch length,

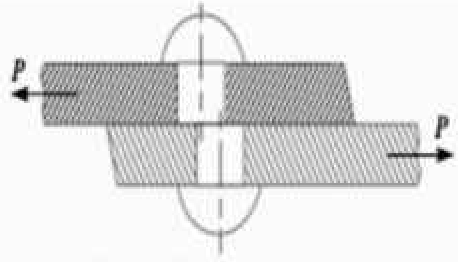
$P_t = A_t . \sigma_t = (p - d)\ t.\ \sigma_t$

When the tearing resistance (P_t) is greater than the applied load (P) per pitch length, then this type of failurewill not occur.

2. Shearing of the rivets:

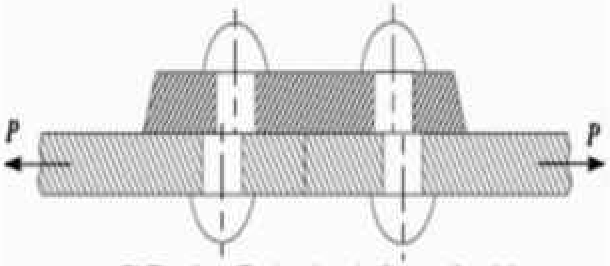
The plates which are connected by the rivets exert shear stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig.

It may be noted that the rivets are in single shear in a lap joint and in a single cover butt joint, as shown in Fig.(a) and (b)

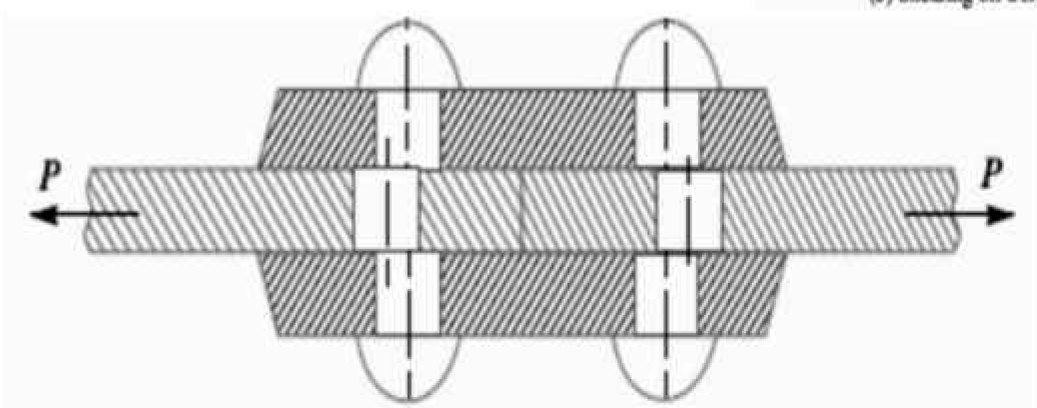


(a) Shearing off a rivet in a lap joint.

But the rivets are in double shear in a double cover butt joint as shown in Fig(c). The resistance offered by a rivet to be sheared off is known as shearing resistance or shearing strength or shearing value of the rivet.



(b) Shearing off a rivet in a single cover butt joint.



Letd = Diameter of the rivet hole,

τ = Safe permissible shear stress for the rivet material,

n = Number of rivets per pitch length.

We know that shearing area,

$A_s = (\pi/4) \times d^2$ ... (In single shear)

$= 2 \times (\pi/4) \times d^2$ ... (Theoretically, in double shear)

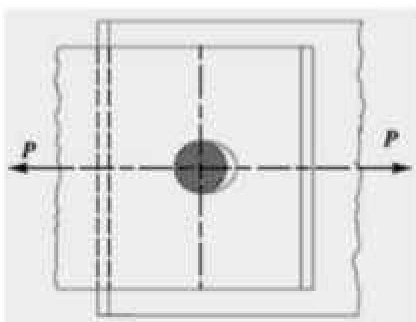
Shearing resistance required to shear off the rivet per pitch length,

$$P_s = n \times (\pi/4) \times d^2 \times \tau \quad \text{...(In single shear)}$$

$$= n \times 2 \times (\pi/4) \times d^2 \times \tau \quad \text{...(Theoretically, in double shear)}$$

When the shearing resistance (P_s) is greater than the applied load (P) per pitch length, then this type of failure will occur.

3. **Crushing of the plate or rivets:** Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as bearing failure. The area which resists this action is the projected area of the hole or rivet on diametral plane.



The resistance offered by a rivet to be crushed is known as crushing resistance or crushing strength orbearing value of the rivet.

Let d = Diameter of the rivet hole,

t = Thickness of the plate,

σ_c = Safe permissible crushing stress for the rivet or plate material,and n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (i.e. projected area per rivet),

$$A_c = d.\ t$$

∴Total crushing area = n. d. t

and crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n.\ d.\ t.\ \sigma_c$$

When the crushing resistance (P_c) is greater than the applied load (P) per pitch length, then this typeof failure will occur.

Note: The number of rivets under shear shall be equal to the number of rivets under crushing.

Unwin's Formula: As a Common Practice for plate thickness greater than 8 mm, the diameter of rivet hole is determined by: $d = 6\sqrt{t}$ (t = thickness of plate)

2.7 Determine strength & efficiency of riveted joints.

Strength of a Riveted Joint:

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail. We have seen that P_t, P_s and P_c are the pulls required to tear off the plate, shearing off the rivet and crushing off the rivet. A little consideration will show that if we go on increasing the pull on a riveted joint, itwill fail when the least of these three pulls is reached, because a higher value of the other pulls will never reach since the joint has failed, either by tearing off the plate, shearing off the rivet or crushing off the rivet.

If the joint is continuous as in case of boilers, the strength is calculated per pitch length.But if the joint is small, the strength is calculated for the whole length of the plate.

Efficiency of a Riveted Joint:

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate. We have already discussed that strength of the riveted joint

$$= \text{Least of } P_t, P_s \text{ and } P_c$$

Strength of the un-riveted or solid plate per pitch length,

$$P = p.\,t\,.\,\sigma_t$$

∴ Efficiency of the riveted joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{p \times t \times \sigma_t}$$

Where, σ_t = Permissible tensile stress of the plate material

p= Pitch of the rivets,

t = Thickness of the plate

Q.1: A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint. If the above joint is subjected to a loadsuch that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution. Given : $t = 15$ mm ; $d = 25$ mm ; $p = 75$ mm ; $\sigma_{tu} = 400$ MPa $= 400$ N/mm^2 ; $\tau_u = 320$ MPa $= 320$ N/mm^2 ; $\sigma_{cu} = 640$ MPa $= 640$ N/mm^2

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)t \times \sigma_{tu} = (75 - 25)15 \times 400 = 300\ 000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u = 2 \times \frac{\pi}{4} (25)^2\, 320 = 314\ 200 \text{ N} \quad ...(\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480\ 000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN. **Ans.**

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300\ 000/4 = 75\ 000 \text{ N}$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75\ 000 = (p - d)\, t \times \sigma_{ta} = (75 - 25)15 \times \sigma_{ta} = 750\, \sigma_{ta}$$

$$\therefore \quad \sigma_{ta} = 75\ 000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Actual shearing resistance of the rivets (P_{sa}),

$$75\ 000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4} (25)^2\, \tau_a = 982\, \tau_a$$

$$\therefore \quad \tau_a = 75000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \textbf{Ans.}$$

and actual crushing resistance of the rivets (P_{ca}),

$$75\ 000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\, \sigma_{ca}$$

$$\therefore \quad \sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Q.2: Find the efficiency of the following riveted joints:

1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm. 2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm. Assume Permissible tensile stress in plate = 120 MPa Permissibleshearing stress in rivets = 90 MPa Permissible crushing stress in rivets = 180 MPa.

Solution. Given : $t = 6$ mm ; $d = 20$ mm ; $\sigma_t = 120$ MPa = 120 N/mm^2 ; $\tau = 90$ MPa = 90 N/mm^2 ; $\sigma_c = 180$ MPa = 180 N/mm^2

1. *Efficiency of the first joint*

Pitch, $p = 50$ mm ...(Given)

First of all, let us find the tearing resistance of the plate, shearing and crushing resistances of the rivets.

(*i*) *Tearing resistance of the plate*

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (50 - 20)\, 6 \times 120 = 21\ 600 \text{ N}$$

(*ii*) *Shearing resistance of the rivet*

Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4}\, (20)^2\, 90 = 28\ 278 \text{ N}$$

(*iii*) *Crushing resistance of the rivet*

Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21\ 600 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 21\ 600 \text{ N}$$

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36\ 000 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{21\ 600}{36\ 000} = 0.60 \text{ or } 60\% \quad \textbf{Ans.}$$

2. *Efficiency of the second joint*

Pitch, $p = 65$ mm ...(Given)

(*i*) *Tearing resistance of the plate,*

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (65 - 20)\, 6 \times 120 = 32\ 400 \text{ N}$$

(*ii*) *Shearing resistance of the rivets*

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}\, (20)^2\, 90 = 56\ 556 \text{ N}$$

(*iii*) *Crushing resistance of the rivet*

Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43\ 200 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 32\ 400 \text{ N}$$

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46\,800 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{32\,400}{46\,800} = 0.692 \text{ or } 69.2\% \quad \textbf{Ans.}$$

Q.3: Design a double riveted lap joint for MS Plates having a thickness 9.5 mm. Calculate the efficiency ofthe joint. The permissible stresses are: σ_t = 90 MPa, τ_s = 75 MPa, σ_c = 150 MPa.

2.8 Design riveted joints for pressure vessel.

Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1. Thickness of boiler shell.

First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, *i.e.*

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance}$$

Where t = Thickness of the boiler shell,

P = Steam pressure in boiler,

D = Internal diameter of boiler

σ_t = Permissible tensile stress, and

η_l = Efficiency of the longitudinal joint.

The following points may be noted:

(a) The thickness of the boiler shell should not be less than 7 mm.

(b) The efficiency of the joint may be taken from the following table.

Indian Boiler Regulations (I.B.R.) allows a maximum efficiency of 85% for the best joint.

(c) According to I.B.R., the factor of safety should not be less than 4.

2. Diameter of rivets.

After finding out the thickness of the boiler shell (*t*), the diameter of the rivet hole (*d*) may be determined by using Unwin's empirical formula,

i.e. $d = 6\sqrt{t}$, (when *t* is greater than 8 mm)

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing.

3. Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that;

(a) The pitch of the rivets should not be less than 2d, which is necessary for the formation of head.

(b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is

$$p_{max} = C \times t + 41.28 \text{ mm}$$

where, t = Thickness of the shell plate in mm,

C = Constant.

- The value of the constant C may be taken from DDB. If the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than p_{max}, then the value of p_{max} is taken.

4. Distance between the rows of rivets.

The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows:

(a) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets (p_b) should not be less than

0.33 p + 0.67 d ......for zig-zig riveting, and

2d ....for chain riveting.

(b) For joints in which the number of rivets in outer rows is **half** the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than **0.33 p + 0.67 or 2d**, whichever is greater.

The distance between the rows in which there are full number of rivets shall not be less than 2d.

(c) For joints in which the number of rivets in outer rows is **half** the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than 0.2 p + 1.15 d. The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than **0.165 p + 0.67d**.

Note : In the above discussion, p is the pitch of the rivets in the outer rows.

5. Thickness of butt strap.

According to I.B.R., the thicknesses for butt strap (t_1) are as given below:

(a) The thickness of butt strap, in no case, shall be less than 10 mm.

(b) t_1 **= 1.125 t**, for ordinary (chain riveting) single butt strap.

$$t_1 = 1.125\, t \left(\frac{p - d}{p - 2d} \right)$$

For single butt straps, every alternate rivet in outer rows being omitted.

t_1 **= 0.625 t**, for double butt-straps of equal width having ordinary riveting (chain riveting).

$$t_1 = 0.625\, t \left(\frac{p - d}{p - 2d} \right)$$

For double butt straps of equal width having every alternate rivet in the outer rows being omitted.

(c) For unequal width of butt straps, the thicknesses of butt strap are

$t_1 = 0.75\, t$, for wide strap on the inside, and

$t_1 = 0.625\, t$, for narrow strap on the outside.

6. Margin.

The margin (m) is taken as 1.5 d.

Design of Circumferential Lap Joint for a Boiler

1. Thickness of the shell and diameter of rivets. The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

2. Number of rivets. Since it is a lap joint, therefore the rivets will be in single shear.

∴ Shearing resistance of the rivets

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \qquad ...(i)$$

where n = Total number of rivets.

Knowing the inner diameter of the boiler shell (D), and the pressure of steam (P), the total shearing load acting on the circumferential joint,

$$W_s = \frac{\pi}{4} \times D^2 \times P \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\therefore \qquad n = \left(\frac{D}{d}\right)^2 \frac{P}{\tau}$$

3.Pitch of rivets. If the efficiency of the longitudinal joint is known, then the efficiency of the circumferential joint may be obtained. It is generally taken as 50% of tearing efficiency in longitudinal joint, but if more than one circumferential joints is used, then it is 62% for the intermediate joints.

Knowing the efficiency of the circumferential lap joint (η_c), the pitch of the rivets for the lap joint.

(p_1) may be obtained by using the relation

$$\eta_c = \frac{p_1 - d}{p_1}$$

The number of rows of rivets for the circumferential joint may be obtained

from the following relation :

4. *Number of rows*. The number of rows of rivets for the circumferential joint may be obtained from the following relation :

$$\text{Number of rows} = \frac{\text{Total number of rivets}}{\text{Number of rivets in one row}}$$

and the number of rivets in one row

$$= \frac{\pi (D + t)}{p_1}$$

where D = Inner diameter of shell.

5. After finding out the number of rows, the type of the joint (*i.e.* single riveted or double riveted etc.) may be decided. Then the number of rivets in a row and pitch may be re-adjusted. In order to have a leak-proof joint, the pitch for the joint should be checked from Indian Boiler Regulations.

6. The distance between the rows of rivets (*i.e.* back pitch) is calculated by using the relations as discussed in the previous article.

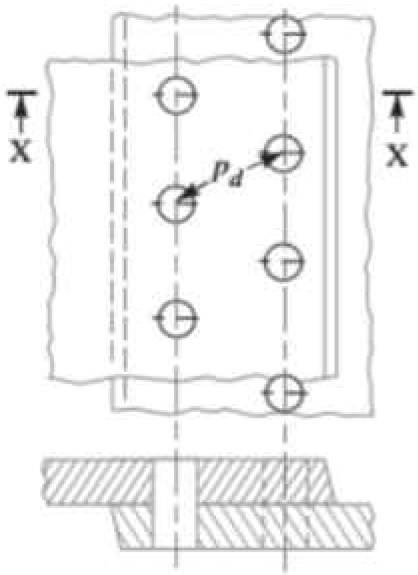
7. After knowing the distance between the rows of rivets (p_b), the overlap of the plate may be fixed by using the relation,

$$\text{Overlap} = (\text{No. of rows of rivets} - 1)\, p_b + m$$

where m = Margin.

2.9 Numerical on Riveted Joints.

Q 4 .A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume σt = 80 MPa ; τ = 60 MPa ; and σc = 120 MPa State how the joint will fail and find the efficiency of the joint



(*c*) **Double riveted lap joint (Zig-zag riveting).**

Solution. Given : t = 13 mm ; σ_t = 80 MPa = 80 N/mm^2 ; τ = 60 MPa = 60 N/mm^2 ; σ_c = 120 MPa = 120 N/mm^2

1. *Diameter of rivet*

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{13} = 21.6 \text{ mm}$$

From Table 9.3, we find that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard size of the rivet hole (d) is 23 mm and the corresponding diameter of the rivet is 22 mm. **Ans.**

2. *Pitch of rivets*

Let p = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting [See Fig.) therefore there are two rivets per pitch length, *i.e.* n = 2. Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)t \times \sigma_t = (p - 23)\, 13 \times 80 = (p - 23)\, 1040 \text{ N} \quad ...(i)$$

and shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (23)^2 \, 60 = 49\,864 \text{ N} \quad ...(ii)$$

...($\because$ There are two rivets in single shear)

From equations (*i*) and (*ii*), we get

$$p - 23 = 49864 / 1040 = 48 \quad \text{or} \quad p = 48 + 23 = 71 \text{ mm}$$

The maximum pitch is given by,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for 2 rivets per pitch length, the value of C is 2.62.

$$\therefore \quad p_{max} = 2.62 \times 13 + 41.28 = 75.28 \text{ mm}$$

Since p_{max} is more than p, therefore we shall adopt

$$p = 71 \text{ mm} \quad \textbf{Ans.}$$

3. *Distance between the rows of rivets*

We know that the distance between the rows of rivets (for zig-zag riveting),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 71 + 0.67 \times 23 \text{ mm}$$
$$= 38.8 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

4. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 23 = 34.5 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

Failure of the joint

Now let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (71 - 23)13 \times 80 = 49\,920 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (23)^2 \, 60 = 49\,864 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71\,760 \text{ N}$$

The least of P_t, P_s and P_c is $P_s = 49\,864$ N. Hence the joint will fail due to shearing of the rivets. **Ans.**

Efficiency of the joint

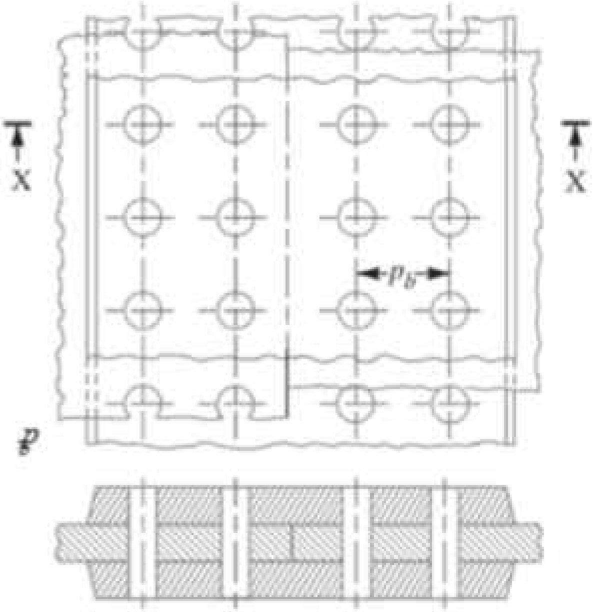
We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73\,840 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{P_s}{P} = \frac{49\,864}{73\,840} = 0.675 \text{ or } 67.5\% \quad \textbf{Ans.}$$

Q 5. Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm 2 . Assume joint efficiency as 75%, allowable tensile stress in the plate 90 MPa ; compressive stress 140 MPa ; and shear stress in the rivet 56 MPa.



(*a*) Chain riveting.

Solution. Given : $D = 1.5$ m = 1500 mm ; $P = 0.95$ N/mm^2 ; $\eta_l = 75\% = 0.75$; $\sigma_t = 90$ MPa = 90 N/mm^2 ; $\sigma_c = 140$ MPa = 140 N/mm^2 ; $\tau = 56$ MPa = 56 N/mm^2

1. ***Thickness of boiler shell plate***

We know that thickness of boiler shell plate,

$$t = \frac{P.D}{2\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12 \text{ mm} \quad \textbf{Ans.}$$

2. ***Diameter of rivet***

Since the thickness of the plate is greater than 8 mm, therefore the diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.8 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 21 mm and the corresponding diameter of the rivet is 20 mm. **Ans.**

3. ***Pitch of rivets***

Let p = Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 21)12 \times 90 = 1080\,(p - 21)\text{N} \quad ...(i)$$

Since the joint is double riveted double strap butt joint, as shown in Fig. therefore there are two rivets per pitch length (*i.e.* $n = 2$) and the rivets are in double shear. Assuming that the rivets in double shear are 1.875 times stronger than in single shear, we have

Shearing strength of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 \text{ N}$$

$$= 72\ 745 \text{ N} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we get

$$1080\,(p - 21) = 72\ 745$$

$$\therefore \quad p - 21 = 72\ 745 / 1080 = 67.35 \text{ or } p = 67.35 + 21 = 88.35 \text{ say } 90 \text{ mm}$$

According to I.B.R., the maximum pitch of rivets for longitudinal joint of a boiler is given by

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for a double riveted double strap butt joint and two rivets per pitch length, the value of C is 3.50.

$$\therefore \quad p_{max} = 3.5 \times 12 + 41.28 = 83.28 \text{ say } 84 \text{ mm}$$

Since the value of p is more than p_{max}, therefore we shall adopt pitch of the rivets,

$$p = p_{max} = 84 \text{ mm} \quad \textbf{Ans.}$$

4. *Distance between rows of rivets*

Assuming zig-zag riveting, the distance between the rows of the rivets (according to I.B.R.),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 84 + 0.67 \times 21 = 41.8 \text{ say } 42 \text{ mm} \quad \textbf{Ans.}$$

5. *Thickness of cover plates*

According to I.B.R., the thickness of each cover plate of equal width is

$$t_1 = 0.625\,t = 0.625 \times 12 = 7.5 \text{ mm} \quad \textbf{Ans.}$$

6. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 21 = 31.5 \text{ say } 32 \text{ mm} \quad \textbf{Ans.}$$

Let us now find the efficiency for the designed joint.

Tearing resistance of the plate,

$$P_t = (p - d)\,t \times \sigma_t = (84 - 21)12 \times 90 = 68\ 040 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 = 72\ 745 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = 70\ 560 \text{ N}$$

Since the strength of riveted joint is the least value of P_t, P_s or P_c, therefore strength of the riveted joint,

$$P_t = 68\ 040 \text{ N}$$

We know that strength of the un-riveted plate,

$$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90\ 720 \text{ N}$$

$\therefore$ Efficiency of the designed joint,

$$\eta = \frac{P_t}{P} = \frac{68\ 040}{90\ 720} = 0.75 \text{ or } 75\% \quad \textbf{Ans.}$$

Chapter-3

DESIGN OF SHAFTS AND KEYS

3.1 State function of shafts.

Shafts:

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) setup within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical,but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used. An **axle,** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave. A **spindle** is a short shaft that imparts motion either to a cutting tool (*e.g.* drill press spindles) or to a work piece (*e.g.* lathe spindles).

Types of Shafts

The following two types of shafts are important from the subject point of view:

Transmission shafts. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bendingin addition to twisting.

Machine shafts. These shafts form an integral part of the machine itself. The crank shaft is an example ofmachine shaft.

3.2 State materials for shafts.

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

When a shaft of high strength is required, then alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

The mechanical properties of these grades of carbon steel are given in the following table.

Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

3.3 Design solid & hollow shafts

Stresses in Shafts

The following stresses are induced in the shafts:

- Shear stresses due to the transmission of torque (*i.e.* due to torsional load).
- Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears,pulleys etc. as well as due to the weight of the shaft itself.
- Stresses due to combined torsional and bending loads.Design of Shafts

The shafts may be designed on the basis of

1. Strength, and

2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment OnlySolid shaft:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

$= d / 2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} d^4$$

Then we get, $$T = \frac{\pi d^3}{16} \tau$$

From this equation, diameter of the solid shaft (d) may be obtained.

Hollow shaft:

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

Where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_0 / 2$.

Substituting these values in equation **(*i*)**, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

Let k = Ratio of inside diameter and outside diameter of the shaft $= d_i / d_o$

Now the equation **(*iii*)** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

From the equations, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

---

2. The twisting moment (T) may be obtained by using the following relation:

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

Where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$T = (T_1 - T_2)\,R$

Where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and R = Radius of the pulley.

Q1.A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Given : $P = 1\text{ MW} = 1 \times 10^6\text{ W}$; $N = 240$ r.p.m. ; $T_{max} = 1.2\,T_{mean}$; $\tau = 60\text{ MPa} = 60\text{ N/mm}^2$

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784\text{ N-m} = 39\ 784 \times 10^3\text{ N-mm}$$

∴ Maximum torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

∴ $d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$

or $d = 159.4$ say 160 mm

Q2 Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; $F.S. = 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

∴ $d^3 = 955 \times 10^3 / 8.84 = 108\ 032$ or $d = 47.6$ say 50 mm.

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\ (d_o)^3\ [1 - (0.5)^4] = 8.3\ (d_o)^3$$

∴ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\ 060$ or $d_o = 48.6$ say 50 mm

$d_i = 0.5\ d_o = 0.5 \times 50 = 25$ mm

Shafts Subjected to Bending Moment Only

a) Solid Shaft:

When the shaft is subjected to a bending moment only, then the maximum stress (tensile orcompressive) is given by the bending equation. We know that

Where M = Bending moment,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

b) Hollow Shaft:

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \text{...(where } k = d_i / d_o)$$

And $y = d_o/ 2$

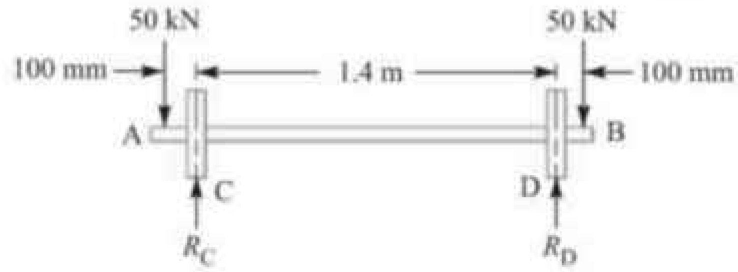
Again substituting these values in equation, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Q3 A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Given : $W = 50\text{ kN} = 50 \times 10^3\text{ N}$; $L = 100\text{ mm}$; $x = 1.4\text{ m}$; $\sigma_b = 100\text{ MPa} = 100\text{ N/mm}^2$



The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$^*M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm Ans.}$$

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must bedesigned on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory: It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory : It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

a) Solid Shaft:

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of σ_b and τ

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

The expression T_e

is known as ***equivalent twisting moment*** and is denoted by T_e.

The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, theequation ***(i)*** may be written as

From this expression, diameter of the shaft (*d)* may be evaluated.

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2} \sigma_b + \frac{1}{2} \sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2} \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

the

The above expression is known as **equivalent bending moment** (M_e).

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3$$

b) Hollow shaft:

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) - \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

It is suggested that diameter of the shaft may be obtained by using both the theories and the larger ofthe two values is adopted.

Q4. A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft

Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm ;
$\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\ d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 86 \text{ say } 90 \text{ mm}$$

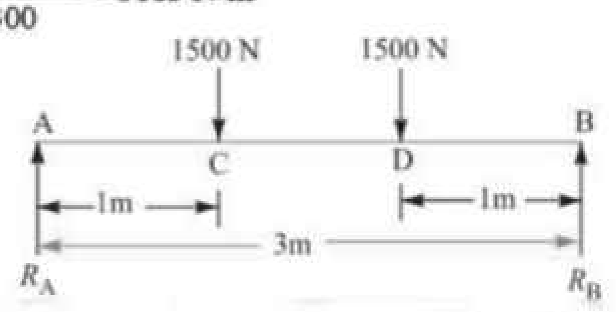
Q 5 A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.

Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*



$$R_A = R_B = 1500 \text{ N}$$

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

∴ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$
$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

∴ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm.

Design of Shafts on the basis of Rigidity:

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. **Torsional rigidity.** The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

where

θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on theshaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. **Lateral rigidity.** It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces.
The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

3.4 State standard size of shaft as per I.S.

The standard sizes of transmission shafts are :

- 25 mm to 60 mm with 5 mm steps;
- 60 mm to 110 mm with 10 mm steps ;
- 110 mm to 140 mm with 15 mm steps ;
- 140 mm to 500 mm with 20 mm steps.
- The standard length of the shafts are 5 m, 6 m and 7 m.

Q 5.A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Solution. Given : $P = 4$ kW $= 4000$ W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m $= 1000$ mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m² $= 84 \times 10^3$ N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or

$$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$\therefore$ $d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\,740 / 8420 = 5.67$ N/mm² $= 5.67$ MPa **Ans.**

Q 6.Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4) \quad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \quad ...(iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3 \, (1 - k^4)}{d^3} = \frac{(d_o)^3 \, (1 - k^4)}{(d_o)^3} = 1 - k^4 \quad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

$\therefore$ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \quad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \quad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Design of keys:

3.5 State function of keys, types of keys & material of keys.

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view:

1. Sunk keys,
2. ***Saddle keys,***
3. Tangent keys,
4. ***Round keys, and***
5. Splines.

Sunk Keys:

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or bossof the pulley. The sunk keys are of the following types :

1. Rectangular sunk key.

A rectangular sunk key is shown in Fig. The usual proportions of this key are :

Width of key, w = d / 4 ; and

thickness of key,

t = 2w / 3 = d/6

Where d = Diameter of the shaft or diameter of the hole in the hub.The key has taper 1 in 100 on the top side only.

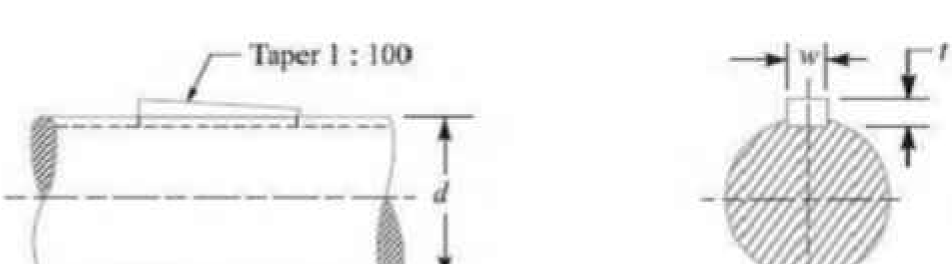


Fig. Sunk Key

2. Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e. w = t = d / 4

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key. It is a rectangular sunk key with a head at one end known as **gib head.** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig.

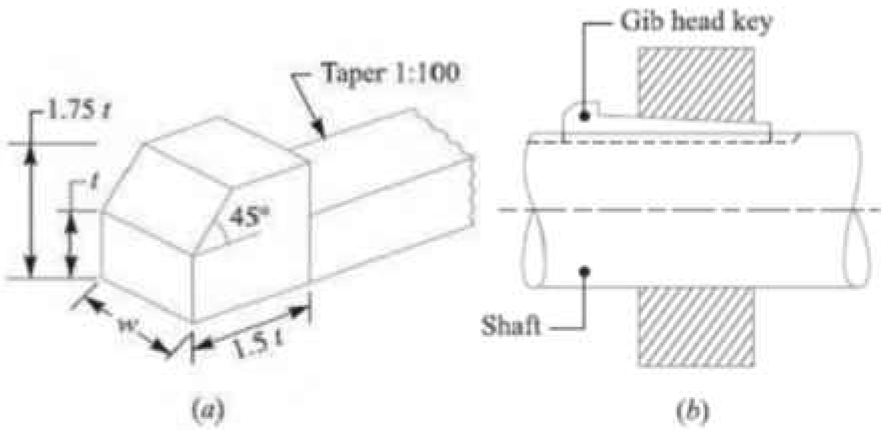


Fig. Gib head key and its use

The usual proportions of the gib head key are:

Width, w = d / 4 ; and

thickness at large end, t = 2w / 3 = d /6.

5. <u>Feather key.</u> A key attached to one member of a pair and which permits relative axial movement is known as **feather key.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, thekey being a sliding fit in the key way of the moving piece.

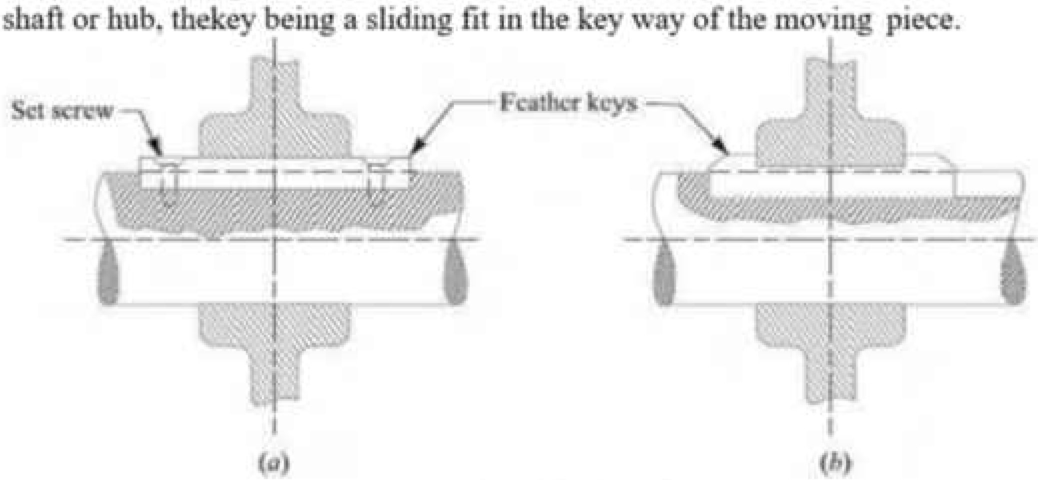


Fig. Feather Keys

6. **Woodruff key.**

The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

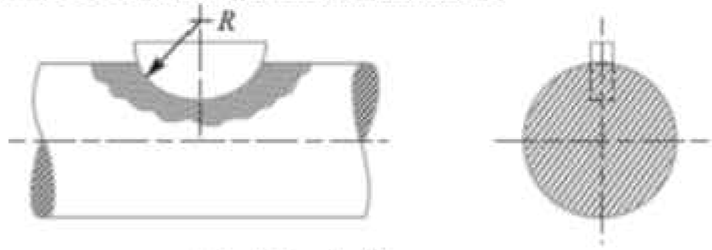


Fig. Woodruff

KeyThe main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.

2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turnover in its keyway.

The disadvantages are:

1. The depth of the keyway weakens the shaft

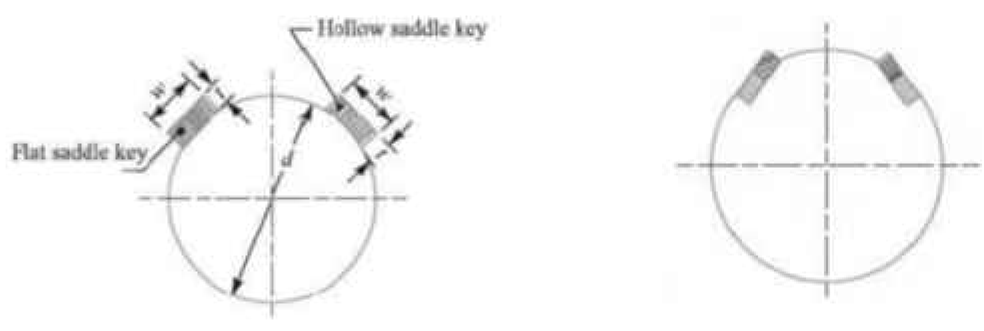
2. It can not be used as a feather.

Saddle keys:

A saddle keys are of the following types

1. flat saddle key

2. hollow saddle key



A **flat saddle key** is a type of key which fits in the hub and is flat on the shaft as shown in fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

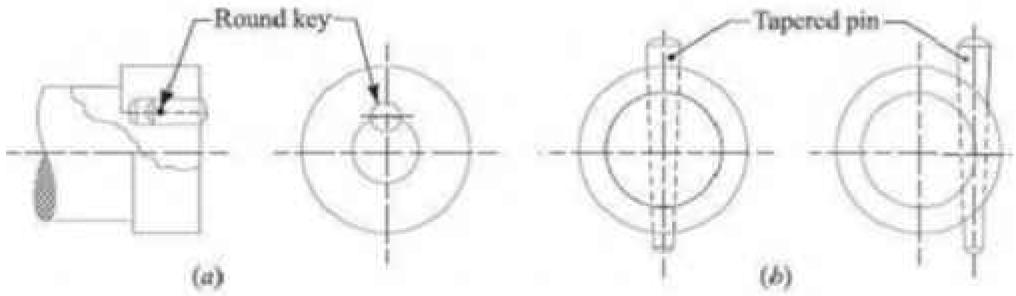
A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

Round Keys

The round keys, as shown in Fig. (a) are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually consideredto be most appropriate for low power drives.



Material Of Keys.

- Typically, shaft keys are made from either medium carbon steel or stainless steel. But they can be made from many different types of material such as aluminium alloy, bronze, copper, and brass to suit different application environments.

3.6 Failure Of Key

Stresses in Keys:

Forces acting on a Sunk Key

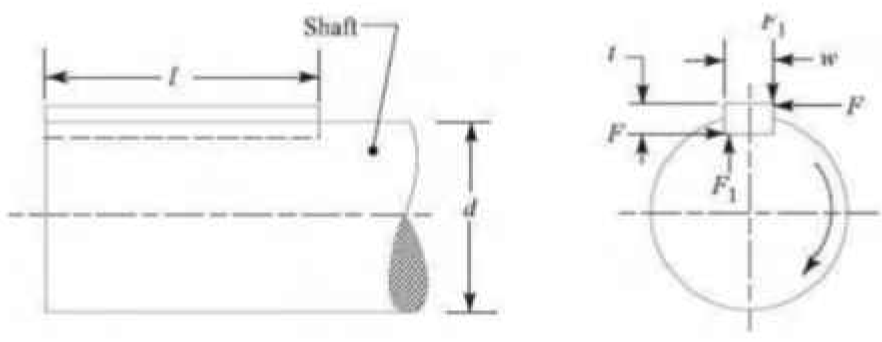
When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub areshown in Fig.

In designing a key, forces due to fit of the key are neglected and it is assumed that thedistribution of forces along the length of key is uniform.



3.7 Design rectangular sunk key considering its failure against shear & crushing

Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig.Let

T = Torque transmitted by the shaft,
F = Tangential force acting at the circumference of the shaft,d = Diameter of shaft,
l = Length of key, w = Width of key.
t = Thickness of key, and
τ and σ_C = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing. Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = l × w × τ

Therefore, Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference

o

fthe shaft,

F = Area resisting crushing × Crushing stress

$$= l \times \frac{t}{2} \times \sigma_c$$

Therefore, Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} - l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

Or

$$\frac{w}{t} - \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the usual key material is at least twice the permissible shearingstress. Therefore from the above equation, we have w = t. In other words, a square key is equally strong in shearing and crushing.
In order to find the length of the key to transmit full power of the shaft, the shearing strength of the keyis equal to the torsional shear strength of the shaft. We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2}$$

And torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3$$

From the above

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$. So, l = 1.571 d.

Effect of Keyway

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = $\frac{\text{Thickness of key } (t)}{2}$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

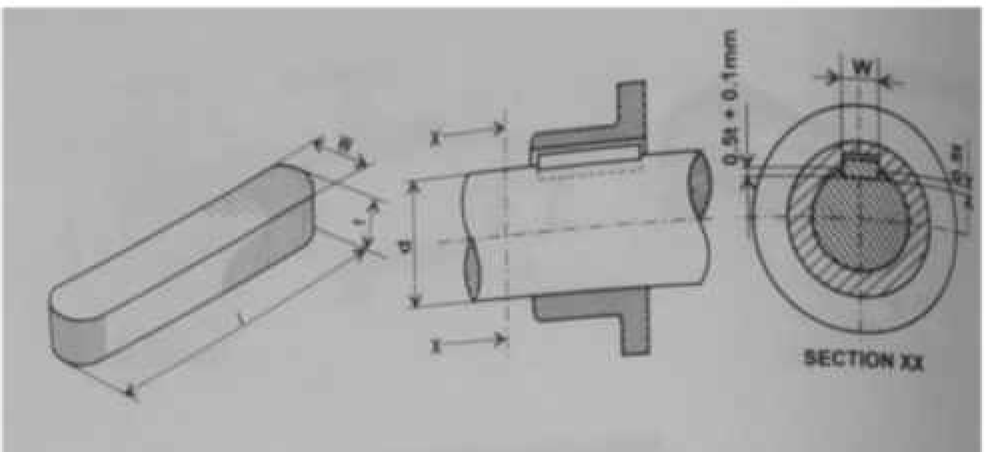
In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

3.9 State specification of parallel key, gib-head key, taper key as per I.S

PARALLEL KEY

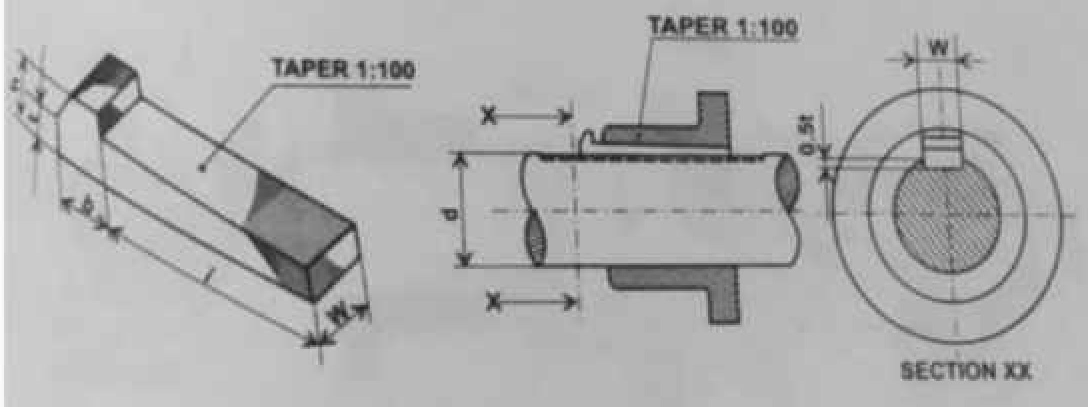


Width of key = 0.25 d + 2 mm

Nominal thickness = 0.66 w

Fig. : Feather or Parallel Key

GIB-HEAD KEY



Width of key = 0.25 d + 2 mm

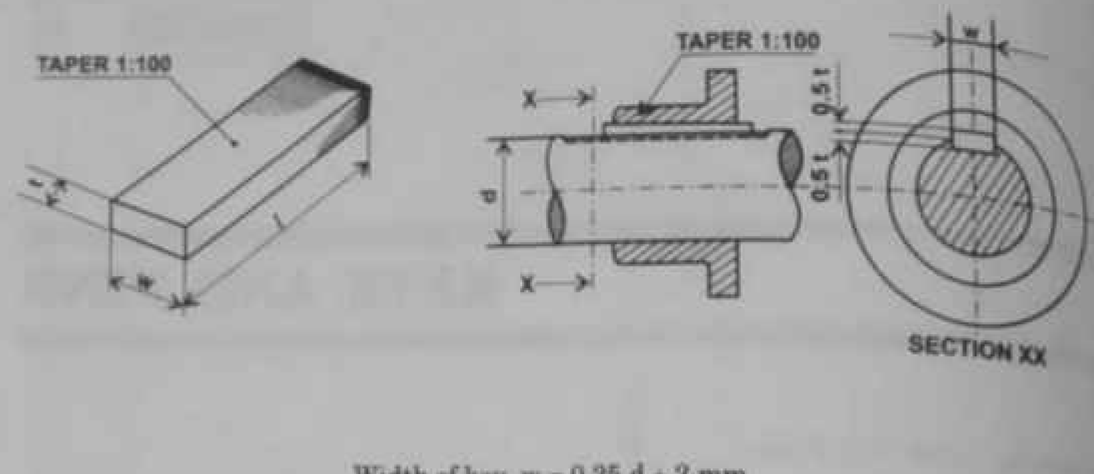
Nominal Thickness = 0.66 w

Standard Taper = 1 : 100

Height of Gib-head = 1.75 t

Width of Gib-head = 1.5 t

TAPER KEY



Width of key, w = 0.25 d + 2 mm

Nominal thickness, T = 0.66 w

Standard taper = 1:100

Fig. Sunk Taper Key

Q1.Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

Given : $d = 50$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_c = 70$ MPa $= 70$ N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm

and thickness of key, $t = 10$ mm

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \quad \textbf{(i)}$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \textbf{(ii)}$$

From equations **(i)** and **(ii)** we have

$l = 1.03 \times 10^6 / 16\,800 = 61.31$ mm

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \quad \textbf{(iii)}$$

From equations **(ii)** and **(iii)**, we have

$l = 1.03 \times 10^6 / 8750 = 117.7$ mm

Taking larger of the two values, we have length of key,

$l = 117.7$ say 120 mm

Q2.A 45 mm diameter shaft is made of steel with yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Given : $d = 45$ mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; $w = 14$ mm ; $t = 9$ mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S. = 2$

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$$\therefore \quad l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\, l$$

$$\ldots\left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$$\therefore \quad l = 1.8 \times 10^6 / 17\,213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

$$l = 104.6 \text{ say } 105 \text{ mm}.$$

Chapter-4

DESIGN OF COUPLING

4.1 Shaft Coupling

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.
Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection of shafts of units those are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts.

4.2 Requirements of a good shaft coupling

A good shaft coupling should have the following requirements:

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts

4.3 Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

1. **Rigid coupling.** It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(a) Sleeve or muff coupling.
(b) Clamp or split-muff or compression coupling, and
(c) Flange coupling.

2. **Flexible coupling.** It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

(a) Bushed pin type coupling,
(b) Universal coupling, and
(c) Oldham coupling.

4.4 Design of Sleeve or Muff-Coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig.

The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

The usual proportions of a cast iron sleeve coupling are as follows:
Outer diameter of the sleeve, **D = 2d + 13 mm**
And length of the sleeve, **L = 3.5 d**
Where, d = diameter of the shaft.

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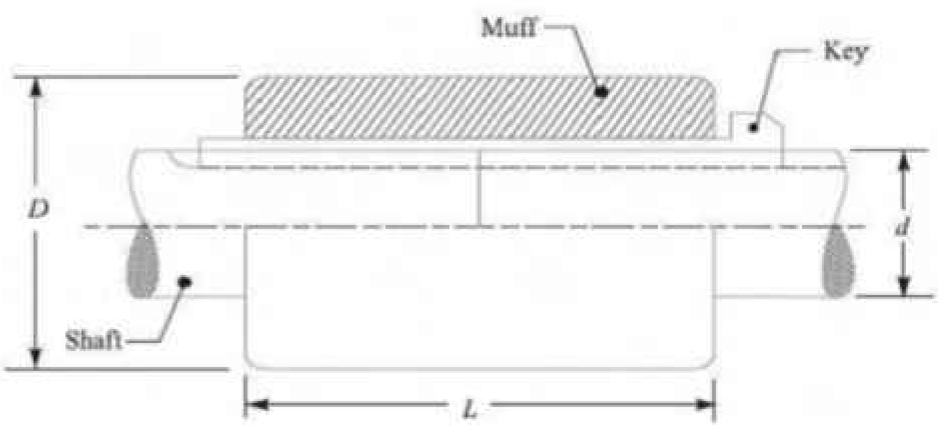


In designing a sleeve or muff-coupling, the following procedure may be adopted

1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft
Let

T = Torque to be transmitted by the coupling, and
τ_c = Permissible shear stress for the material of the sleeve which is cast iron.

The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \qquad \dots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked

2. Design for key

The key for the coupling may be designed in the similar way as discussed in pervious chapter.

The width and thickness of the coupling key is obtained from the proportions.
The length of the coupling key is at least equal to the length of the sleeve (i.e. 3.5 d).
The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Note: The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, then the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, the key is made in two parts which may be driven from the same end for each shaft or they may be driven from opposite ends.

Q1.Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution:
Given: $P = 40$ kW $= 40 \times 10^3$ W; $N = 350$ r.p.m.; τs = 40 MPa = 40 N/mm2; σcs = 80 MPa = 80 N/mm^2; σc = 15 MPa = 15 N/mm^2.

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\,d^3$$

$\therefore \quad d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

1. Design for sleeve

We know that outer diameter of the muff,
D = 2d + 13 mm = 2 × 55 + 13 = 123 say 125 mm **Ans.**

Length of the muff,
L = 3.5 d = 3.5 × 55 = 192.5 say 195 mm **Ans.**

Let us now check the induced shear stress in the muff.

Let τc be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft,

Therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 103\ \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm2, therefore the design of muff is safe.

3. Design for key

From Design data Book, we find that for a shaft of 55 mm diameter,
Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.
Then, Thickness of key, t = w = 18 mm **Ans.**

We know that length of key in each shaft,
l = L / 2 = 195 / 2 = 97.5 mm **Ans.**

Let us now check the induced shear and crushing stresses in the key.

First of all, let us consider shearing of the key.

We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key.

We know that torque transmitted (*T*),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3\ \sigma_{cs}$$

$$\sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

4.5 Design of Clamp or Compression Coupling.

It is also known as **split muff coupling.** In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron.

The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings.

This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling.

The usual proportions of the muff for the clamp or compression coupling are: Diameter of the muff or sleeve,

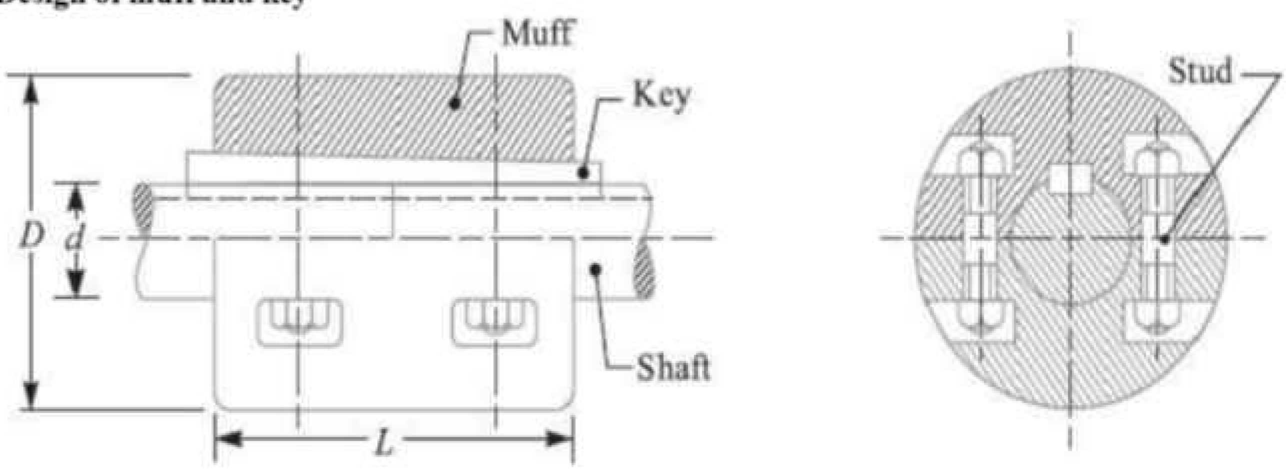
$$D = 2d + 13 \text{ mm}$$

Length of the muff or sleeve, $L = 3.5\ d$

Where d = Diameter of the shaft.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. Design of muff and key



The muff and key are designed in the similar way as discussed in muff coupling.

2. Design of clamping bolts

Let T = Torque transmitted by the shaft,
d = Diameter of shaft,
d_b = Root or effective diameter of bolt, n = Number of bolts,
σt = Permissible tensile stress for bolt material,
μ = Coefficient of friction between the muff and shaft, and
L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

Then, Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,
Then, Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2}\pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

And the torque that can be transmitted by the coupling,
From this relation, the root diameter of the bolt (d_b) may be evaluated

Q2.Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. Design for shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm

2. Design for muff

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm}$$

3. Design for key

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, w = 22 mm

Thickness of key, t = 14 mm

and length of key = Total length of muff = 262.5 mm

4. Design for bolts

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2\, \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\ 70 \times 6 \times 75 = 5830 (d_b)^2$$

$\therefore$ $(d_b)^2 = 2865 \times 10^3 / 5830 = 492$ or $d_b = 22.2$ mm

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27).

Chapter-5

DESIGN OF CLOSED COIL HELICAL SPRING

Spring:

Spring is defined as an elastic machine element (flexible element) that deflects under the action of load and returns to its original shape when load is removed.

Important functions and applications of spring :

1. Springs are used to absorb shocks and vibrations eg: vehicle suspension springs, railway buffers tocontrol energy, buffer springs in elevators and vibration mounts for machinery.
2. Measuring forces : Spring balances, gages
3. Storing of energy in clocks ,toys ,cameras, circuit breakers ,starters
4. Springs are used to apply force and control motion.

5.1 Materials used for helical spring.

One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

Hard-drawn wire: This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 120 ^{0} C.

Oil-tempered wire:

It is a cold drawn, quenched, tempered, and general purpose spring steel. However, it is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 180 ^{0} C. When we go for highly stressed conditions then alloy steels are useful.

Chrome Vanadium:

This alloy spring steel is used for high stress conditions and at high temperature up to 220 ^{0} C. It is good for fatigue resistance and long endurance for shock and impact loads.

Chrome Silicon:

This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250 ^{0} C.

Music wire:

This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. However, it cannot be used at subzero temperatures or at temperatures above 120 ^{0} C. Normally when we talk about springs we will find that the music wire is a common choice for springs.

Stainless steel:

Widely used alloy spring materials.

Phosphor Bronze / Spring Brass:

It has good corrosion resistance and electrical conductivity. That's the reason it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

5.2 Standard size spring wire. (SWG)

British Standard Wire Gauge (often abbreviated to **Standard Wire Gauge or SWG**) is a unit for denoting wire size given by BS 3737:1964. It is also known as the Imperial Wire Gauge or British Standard Gauge.

Standard Wire Gauge (SWG) a notation used for the diameters of metal rods or thickness of metal.

Standard Size of Spring Wire

The standard size of spring wire may be selected from the following table :

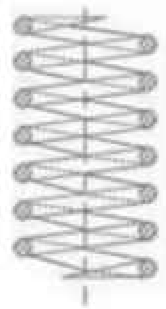
Standard wire gauge (SWG) number and corresponding diameter of spring wire.

SWG	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

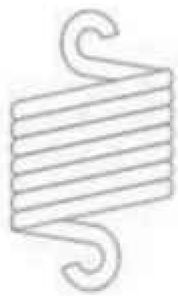
5.3 Terms used in compression spring.

Helical springs.

The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring as shown in Fig (a) and tension helical spring as shown in Fig (b)



(*a*) Compression helical spring.



(*b*) Tension helical spring.

The helical springs are said to be closely coiled when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In open coiled helical springs, the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(a) These are easy to manufacture.
(b) These are available in wide range.
(c) These are reliable.
(d) These have constant spring rate.
(e) Their performance can be predicted more accurately.
(f) Their characteristics can be varied by changing dimensions.

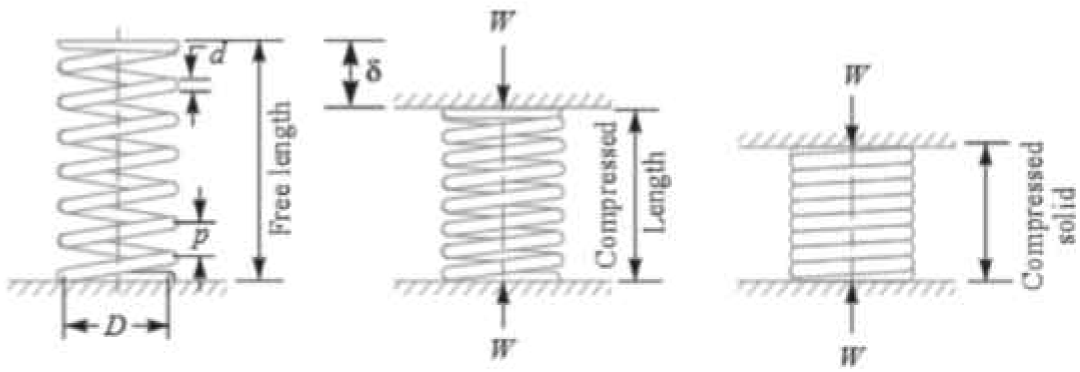
SOLID LENGTH:

When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire.

$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.



FREE LENGTH:

The free length of a compression spring, as shown in Fig, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).

Mathematically,

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

SPRING INDEX:

The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, C = D / d

Where, D = Mean diameter of the coil, and d = Diameter of the wire.

SPRING RATE:

The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring.

Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

PITCH :

The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $$p = \frac{L_F - L_S}{n'} + d$$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

STRESSES IN HELICAL SPRING :

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shownin Fig.

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

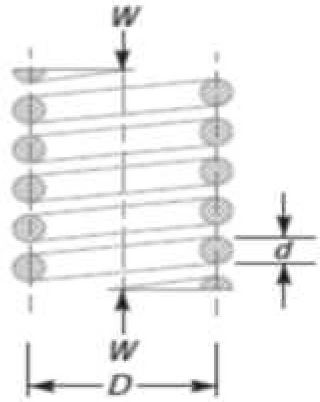
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

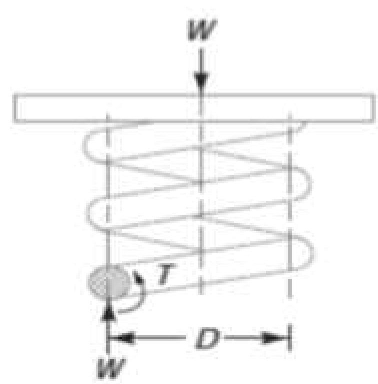
C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.



(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig (b).The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire. A little consideration will show that part of the Spring,as shown in Fig (b), is in equilibrium under the action of two forces W and the twisting moment T.

We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi d^3} \quad ...(i)$$

The torsional shear stress diagram is shown in Fig (a).

In addition to the torsional shear stress ($\tau 1$) induced in the wire, the following stresses also act on the wire:

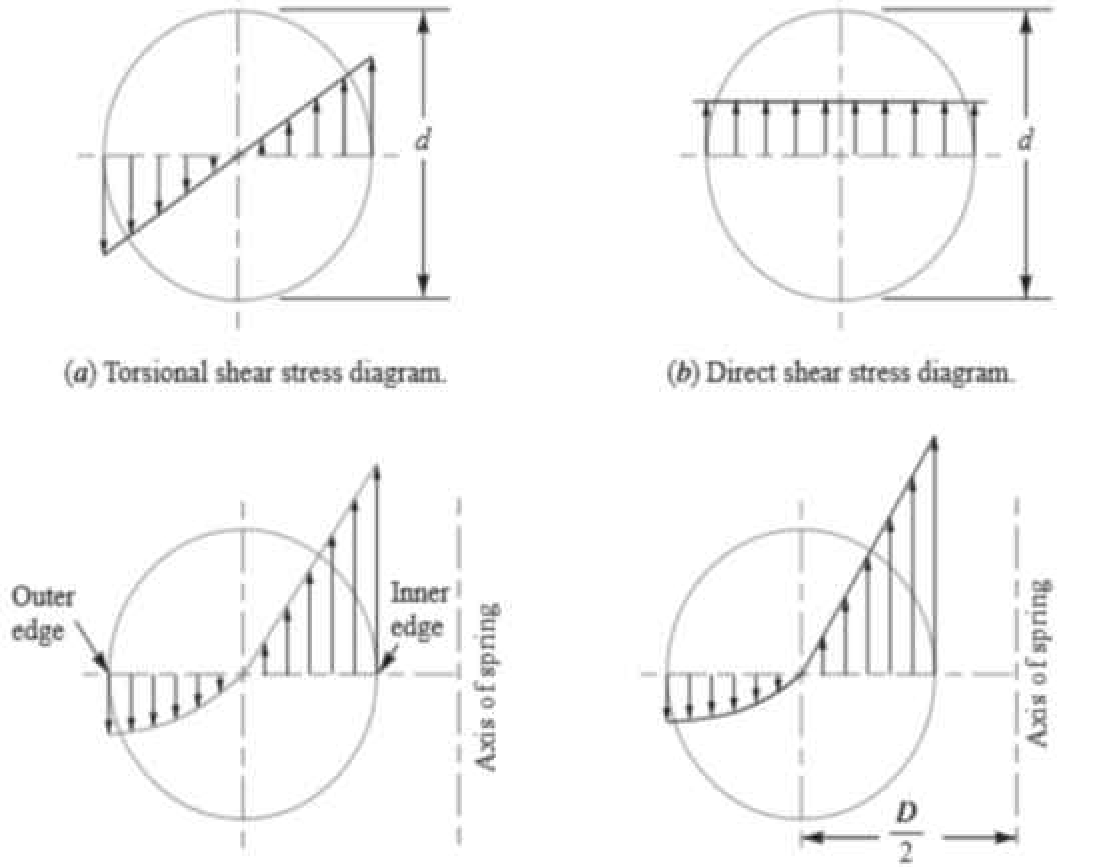
1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \quad ...(ii)$$

The direct shear stress diagram is shown in Fig. (b) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig (c).



(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8\,WD}{\pi\,d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation *(iii)*. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. *(d)*.

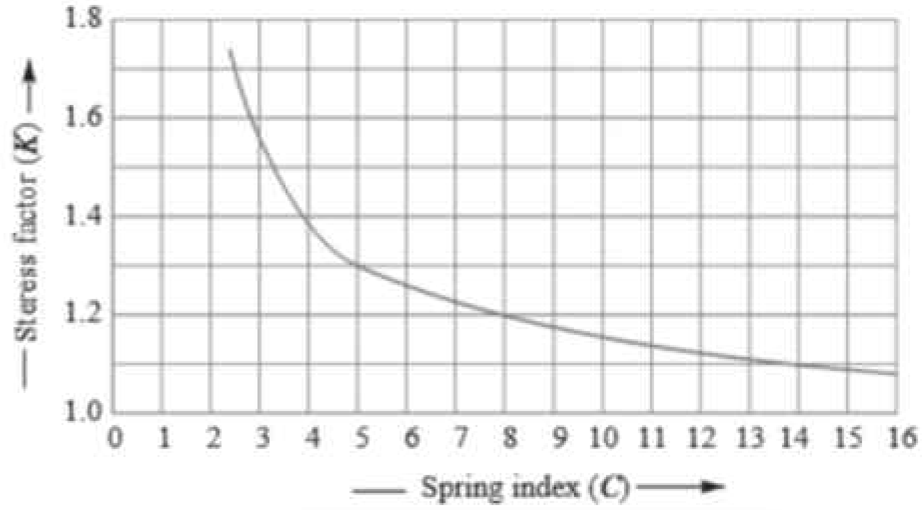
∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \quad ...(iv)$$

where

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig



Wahl's stress factor for helical springs.

We see from Fig. that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that

$$K = K_S \times K_C$$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

5.5 Deflection of helical spring of circular wire.

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T\,l}{J.G} \qquad ...\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T\,l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\, D.n}{\frac{\pi}{32} \times d^4\, G} = \frac{16 W.D^2.n}{G.d^4} \qquad \text{...(ii)}$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \text{... } (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

5.6 Surge in spring

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire.

In the beginning, the end coils of the spring in contact with the applied load take up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions; it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi\, D^2.n}\sqrt{\frac{6\, G.g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

NUMERICALS:

Q1. Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is84 kN/mm 2.

Take Wahl's factor, $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$, *where C = Spring index.*

Solution. Given : W = 1000 N ; δ = 25 mm ; $C = D/d = 5$; τ = 420 MPa = 420 N/mm^2 ; G = 84 kN/mm^2 = 84 × 10^3 N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.31 \times \frac{8\times1000\times5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

$$\therefore \quad d^2 = 16\,677 / 420 = 39.7 \quad \text{or} \quad d = 6.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

∴ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm } \textbf{Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$$\therefore \quad n = 25 / 1.86 = 13.44 \text{ say } 14 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

Q2 A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm2, find the axial load which the spring can carry and the deflection per active turn.

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm^2; $G = 84$ kN/mm^2 $= 84 \times 10^3$ N/mm^2

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$$\therefore \text{ Spring index,} \quad C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$$\therefore \quad W = 350 / 0.848 = 412.7 \text{ N } \textbf{Ans.}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$$\therefore \quad W = 350 / 0.913 = 383.4 \text{ N } \textbf{Ans.}$$

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$