Study Material

On

Fluid Mechanics

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PROPERTIES OF FLUIDS

The study of fluid at rest – Fluid Static

The study of fluid in motion – where pressure forces are not considered – Fluid Kinematics

The study of fluids in motion, where pressure forces are also considered – Fluid Dynamics.

Properties of Fluids

Density or Mass Density – It is the ratio of mass of a fluids to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by symbol ρ (rho). Density of ~~gasses~~ liquids considered constant, while that of gasses varies with temperature and pressure.

$$\rho = \frac{\text{Mass of Fluid}}{\text{Volume of Fluid}}$$

Unit of ρ = Kg/m^3 in S.I.

Density of water = 1 gm/cm^3 or 1000 Kg/m^3

Specific wt. or Weight Density

It is the ratio between the weight of a fluid to its volume. It is denoted by symbol 'w'

$$w = \frac{\text{Weight of Fluid}}{\text{Volume of Fluid}} = \frac{(\text{Mass of fluids}) \times \text{Accelaration due to gravity}}{\text{Volume of fluids}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}} = \rho \times g$$

$$\therefore \quad w = \rho \times g$$

Weight Density = Mass Density x Accn. Due to Gravity

Weight Density of Water = 9.81×1000 $Newton/m^3$ in S.I Unit

Specific Volume – The ~~to~~ Specific Volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called Specific Volume

$$\text{Specific Volume} = \frac{\text{Volume of a fluid}}{\text{Mass of a fluid}}$$

$$= \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of fluid}}} = \frac{1}{\rho}$$

Thus Specific Volume is the reciprocal of mass density. Its unit is m^3/Kg. It is commonly applied to gasses.

Specific Gravity – The specific Gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken as water and for gasses, the standard fluid is taken as air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol 'S'

$$S \text{ (for liquids)} = \frac{\text{Weight density (or density) of liquids}}{\text{Weight density (or density) of Water}}$$

$$S \text{ (for gases)} = \frac{\text{Weight density (or density) of gases}}{\text{Weight density or density of air}}$$

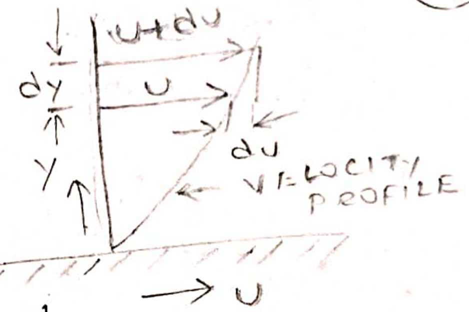
Thus Weight density of liquids $= S \times$ density of Water

~~Weight density of gas~~ $= S \times 1000 \times 9.81 \ N/m^3$

Density of liquid $= S \times$ Density of water

$= S \times 1000 \ Kg/m^3$

VISCOSITY



Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer. When two layers of a fluid, a distance '~~dy~~' apart move one over the other at different velocities say u and u+du, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer and the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to 'y'. It is denoted by 'τ' called Tau

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$, μ = (mu) called is the constant of proportionality and is known as the co-efficient of dynamic viscosity. $\frac{du}{dy}$ is rate of shear strain or rate of shear deformation or velocity gradient.

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$, Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

Units of viscosity

$$\mu = \frac{\text{Shear stress}}{\frac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\frac{\left(\frac{\text{Length}}{\text{time}}\right)}{\text{Length}}}$$

$$= \frac{\frac{\text{Force}}{\text{Area}}}{\frac{\text{Length}}{\text{time}} \times \frac{1}{\text{Length}}} = \frac{\text{Force}}{\text{Length} \times \text{Length}} \times \text{time}$$

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$$= \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

M.K.S Unit $= \frac{\text{Kgf - Sec}}{m^2}$

CGS unit $= \frac{\text{dyne - Sec}}{Cm^2}$

S I Unit $= \frac{\text{Newton - Sec}}{m^2} = \frac{\cancel{N\text{-}Sec}}{\cancel{m^2}}\ \frac{N.S}{m^2}$

The unit of viscosity in CGS is also called **poise** which is equal to $\frac{\text{dyne - sec}}{cm^2}$

One $\frac{\text{Kgf - sec}}{m^2} = \frac{9.81\ \text{N-Sec}}{m^2}$ $\quad (\because 1\ \text{Kgf} = 9.81\ \text{Newton})$

1 Newton $=$ One Kg (mass) $\times$ One $\left(\frac{m}{Sec^2}\right)$ (acceleration)

$$= \frac{(1000\ gm) \times 100\ Cm}{Sec^2} = 1000 \times 100\ \frac{gm\text{-}Cm}{Sec^2}$$

$$= 1000 \times 100\ \text{dyne} \quad \left(\because 1\ \text{dyne} = gm \times \frac{cm}{Sec^2}\right)$$

One $\frac{\text{Kgf - Sec}}{m^2} = 9.81 \times \cancel{1000}\ 1000000\ \frac{\text{dyne - sec}}{m^2}$

$$= 9.81 \times 1000000\ \frac{\text{dyne - Sec}}{100 \times 100\ Cm^2}$$

$$= 9.81 \times 10\ \frac{\text{dyne - Sec}}{Cm^2} = \underline{98.1\ \text{Poise}}$$

Hence if viscosity is given in poise, it must be divided by 98.1 to get the viscosity in M.K.S

One $\frac{\text{Kgf - Sec}}{m^2} = \cancel{98}\ \frac{9.81\ NS}{m^2} = 98.1$ poise

One $\frac{\text{N-sec}}{m^2} = \frac{98.1\ \text{Poise}}{9.81} = 10$ poise

Or, One poise $= \frac{1}{10}\ \frac{N\text{-}S}{m^2}$

If viscosity is given in poise, it should be divided by 10 to get viscosity in S.I unit

Centi Poise = $\frac{1}{100}$ poise

Viscosity of water at 20°C is 0.01 Poise or 1.0 centipoise

Kinematic Viscosity

It is the ratio between dynamic viscosity and density of fluid. It is denoted by Greek symbol 'ν' called 'nu'. (ν)

$$nu = \nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

Units of Kinematic Viscosity

$$\nu = \frac{\text{Units of } \mu}{\text{Unit of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2} \div \frac{\text{Mass}}{(\text{Length})^3}$$

$$= \frac{\text{Force} \times \text{Time}}{(\text{Length})^2} \times \frac{(\text{Length})^3}{\text{Mass}}$$

$$= \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}} = \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

($\because$ Force = Mass × Accln.)

$$= \frac{(\text{Length})^2}{\text{Time}}$$

M.K.S unit → $\frac{\text{meter}^2}{\text{Sec}}$ or $\frac{m^2}{\text{Sec}}$

CGS → $\frac{cm^2}{\text{Sec}} = \boxed{\text{STOKE}}$

Thus 1 Stoke = 1 $\frac{cm^2}{\text{Sec}} = \frac{\left(\frac{1}{100}\right)^2 m^2}{\text{Sec}}$

$= 10^{-4} \frac{m^2}{\text{Sec}}$

CENTI STOKE = $\frac{1}{100}$ Stoke.

Newton's Law of Viscosity

The shear stress 'τ' on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient viscosity. Mathematically

$$\tau = \mu \frac{du}{dy}$$

The fluids which obey the above rel relation known as Newtonian Fluid. are which do not obey the relation are ~~known~~ known as Non-newtonian Fluid.

Variation of viscosity with Temperature

Temp. affects the viscosity. The viscosity of liquid decrease with the increase of temperature while the viscosity of gases increase with increase of temperature. This is due to the reason that the ~~d~~ viscous forces forces in a fluid are due to cohesive forces and molecular momentum transfer. In liquids the cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with increase with temp. the cohesive forces decreases with decrease of viscosity. But in the case of gases the cohesive forces are small and the ~~molecule~~

molecular momentum transfer predominate. 7

With increase in temperature molecular momentum transfer increase and hence viscosity increases.

The relation between viscosity and temperature for liquids and gases are

(i) <u>For Liquids</u> $\mu = \mu_0 \left(\frac{1}{1+\alpha t+\beta t^2} \right)$ ...(1.4A)

μ = Viscosity of liquids at $t^\circ C$ in Poise

μ_0 = Viscosity of liquid at $0^\circ C$ in poise.

α and β are constants for the liquid

For water $\mu_0 = 1.79 \times 10^{-3}$ poise

$\alpha = 0.03368$ and $\beta = 0.000221$

The equation (1.4A) shows that with the increase of temperature, the viscosity ~~increases~~ decreases.

(ii) <u>For Gas</u>

$\mu = \mu_0 + \alpha t - \beta t^2$ --- (1.4B)

for air $\mu_0 = 0.000017$

$\alpha = 0.000000056$

$\beta = 0.1189 \times 10^{-9}$

Eqn. (1.4B) shows that with increase of temperature viscosity increases.

<u>Types OF Fluid</u> – Fluids divided into 6 types.

1. Ideal Fluid , 2 Real Fluids.
3. Newtonian Fluid 4. Non-Newtonian Fluid
5. Ideal Plastic Fluid, 6 - Thixotropic fluid

<u>1. IDEAL FLUID</u> – A fluid which is incompressible and is having no viscosity is known as Ideal Fluid. Ideal

1. Fluid is only a theoretical fluid as all fluids have some viscosity.

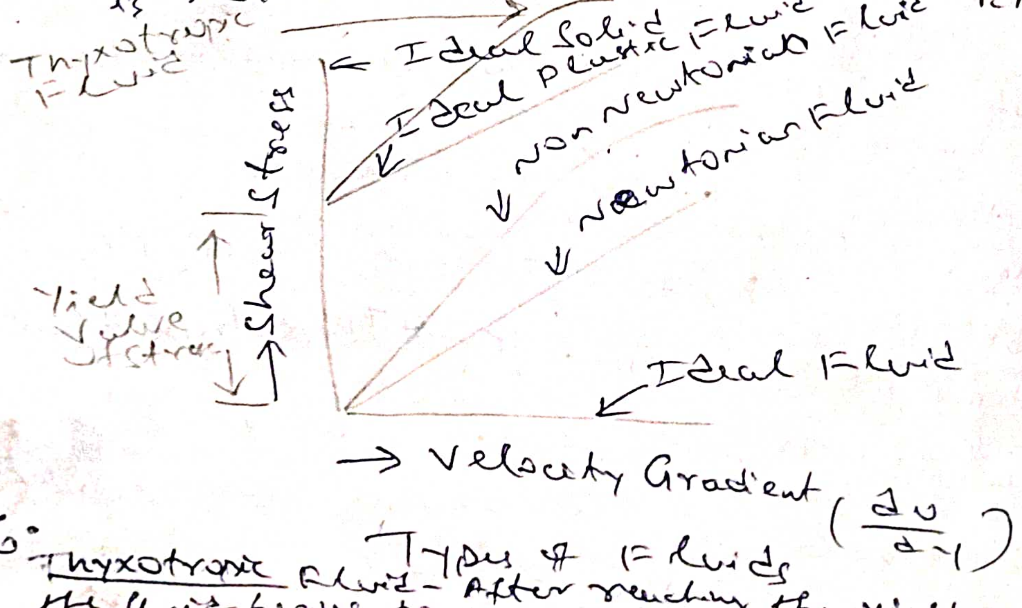
2. REAL FLUID. A fluid which ~~containing~~ ~~possess~~ ~~viscosity~~ possesses viscosity is Real Fluid. All the actual fluids are Real Fluids.

3. NEWTONIAN FLUID – A real fluid which obeys Newton's law of viscosity i.e in which shear stress is directly proportional to the rate of shear strain (or velocity gradient) is ~~also~~ known as Newtonian fluid. Ex – Water, Kerosene, Air

4. NON NEWTONIAN FLUID – The fluid which do not obey the Newton's law of viscosity is Non-Newtonian fluid. Shear stress and shear strain relation is not linear. Ex – Solutions or suspensions, slurries, mud flow, blood

5. Ideal Plastic Fluid

A fluid where shear stress is more than the yield value and shear stress is proportional to the rate of shear strain is known as ideal plastic fluid. Ex – Sewage sludge, Drilling muds



Types of Fluids

6. Thyxotropic Fluid – After reaching the yield value of stress the fluid begins to flow. But relation between shear stress & shear strain ~~is not~~ linear. Ex – Pri

Prob: (1·3) If the velocity distribution over a plate is given by $U = \frac{2}{3}y - y^2$ in which U is the velocity in meter per second at a distance y meter above the plate, determine the shear stress at $y=0$ and $y = 0.15$ m. Take dynamic viscosity of fluid as 8·63 poise.

Ans

$$U = \frac{2}{3}y - y^2 \quad , \therefore \frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{at\ y=0} = \frac{2}{3} - 2\times 0 = \frac{2}{3} = 0.667$$

$$\left(\frac{du}{dy}\right)_{at\ y=0.15} = \frac{2}{3} - 2\times 0.15 = 0.667 - 0.30$$
$$= 0.367$$

$$\mu = 8.63 \text{ poise} = \frac{8.63}{10} \text{ S.I unit} = 0.863 \ \frac{Ns}{m^2}$$

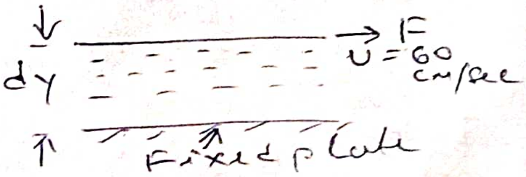
Shear stress $\tau = \mu \frac{du}{dy}$

at $y = 0$ $\quad \tau = 0.863 \times 0.667 = 0.5756 \ N/m^2$

at $y = 0.15$ m $\quad \tau = 0.863 \times 0.367 = 0.3167 \ N/m^2$

Prob: (1·4) A plate 0·025 mm distant from a fixed plate, moves at 60 ~~m/sec~~ cm/sec and requires a force of 2 N per unit area i.e $2 N/m^2$ to maintain this speed. Determine the ~~fixed~~ fluid viscosity between the plate

Soln.



Distance between plates

$dy = 0.025 \text{ mm} = 0.025 \times 10 = 2.5 \times 10^{-5}$ mtr

Velocity of upper plate $= 60$ cm/sec $= 0.6$ m/sec.

Force required to move the plate $= 2 \ N/m^2$

This is shear stress τ

Let the viscosity of fluid = μ

$$\tau = \mu \frac{du}{dy}$$

du = change in velocity = $U - 0 = 0.6$ m/sec

dy = change of distance = 0.025 mm

$= 2.5 \times 10^{-5}$ mtr

~~$\tau = 2.0 \times \frac{0.6}{2.5 \times 10^{-5}}$~~

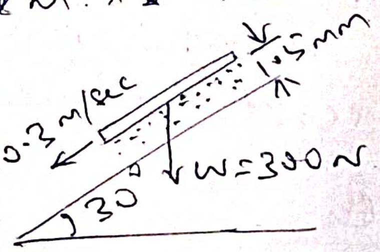
$\tau = 2$ N/m^2

$\therefore \quad 2 = \mu \frac{0.6}{2.5 \times 10^{-5}}$

$\mu = \frac{2 \times 2.5 \times 10^{-5}}{0.6} = 8.333 \times 10^{-5} \frac{Ns}{m^2}$

$= 8.33 \times 10^{-5} \times 10$ poise $= 8.33 \times 10^{-4}$ poise.

Prob: 1.7. Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in figure. The weight of square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/sec. The thickness of oil film is 1.5 mm



Soln.

Thickness of oil film = dy = 1.5 mm

$= 1.5 \times 10^{-3}$ mtr

Let the viscosity of oil = μ

Component of wt along the inclined plane

$= W \sin 30°$

$= 300 N \sin 30° = 150 N$

This is the shear force acting on the plate

$\therefore$ Shear stress $= \frac{150 N}{Area} = \frac{150}{0.8 \times 0.8} N/m^2$

$$\tau = \mu \frac{du}{dy}$$

du = change in velocity $= \cancel{\phi} U - 0$
$= 0.3\ m/sec - 0 = 0.3\ m/sec.$

$dy = 1.5\ mm = 1.5 \times 10^{-3}$ mtr

$$\therefore \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

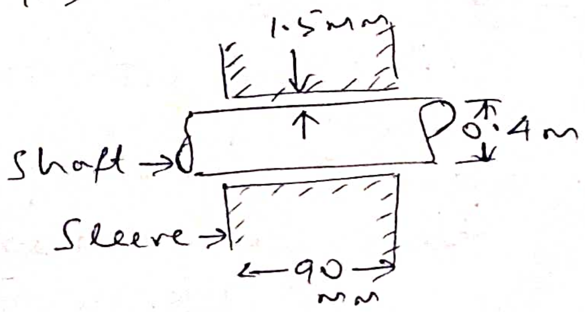
$$\mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.171875 \frac{N.s}{m^2}$$

$= 1.171875 \times 10$ poise $= 11.71875$ poise

$1 \frac{N.s}{m^2} = 10$ poise

Ans.

Prob. 1.4 – The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.



Soln.

$\mu = \cancel{6\ poise}$ 6 poise
$= \frac{6}{10} \frac{N.s}{m^2}$

Dia of shaft $= 0.4$ mtr

Speed of shaft $= 190$ rpm

Sleeve length $= L = 90$ mm $= 90 \times 10^{-3}$ mtr

Thickness of Film $= 1.5$ mm $= 1.5 \times 10^{-3}$ mtr $= dy$

Tangential Velocity $U = \frac{\pi D N}{60}$

$= \frac{\pi \times 0.4 \times 190}{60} = 3.98$ mtr/sec

$\tau = \mu \frac{du}{dy}$

du = change in Velocity $= U - 0 = U = 3.98$ m/sec

dy = change in distance $= 1.5 \times 10^{-3}$ mtr

$\therefore \tau = 0.6 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592$ N/m²

Shear Force $= \tau \times$ Area of ~~Sleeve~~ shaft

$= 1592 \text{ N/m}^2 \times \pi D \times L$

$= 1592 \times \pi \times 0.4 \times 90 \times 10^{-3}$

$= 180.0509$ N

Torque of shaft $=$ Force $\times \frac{D}{2}$

$= 180.0509 \times \frac{0.4}{2} = 36.01$ N-m

Power Lost $= \frac{2 \pi N T}{60} = \frac{2\pi \times 190 \times 36.01}{60}$

$= 716.48$ Watt

$\because$ Power in S-I Unit $= T \times \omega$

$= T \times \frac{2\pi N}{60}$ Watt

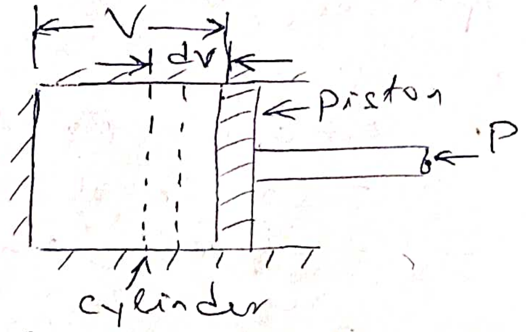
(Also power = Force $\times$ Velocity

$= 180.0509 \text{ N} \times 3.98$ m/sec

$= 716.60$ Watt)

COMPRESSIBILITY AND BULK MODULUS

Compressibility is the reciprocal of bulk modulus of elasticity 'K' which is defined as the ratio of compressive stress to volumetric strain.



Consider a cylinder fitted with a piston

Let V = Volume of gas enclosed in the cyl.

P = pressure of gas when vol = V

Let the pressure is increased to P + dp

Vol. of gas decreased to V − dv

Hence increase in pressure = dp kgf/m^2

decrease in vol = dv

Volumetric strain $= -\dfrac{dv}{V}$

− ve sign indicates the vol decrease with increase of pressure

∴ Bulk modulus, $K = \dfrac{\text{Comp. stress}}{\text{volumetric strain}}$

$$= \frac{\text{Increase of Pressure}}{\text{Volumetric strain}} = \frac{dp}{-\frac{dv}{V}}$$

$$K = -\frac{dp}{dV} \times V \quad \text{----(1.10)}$$

[Comp. stress = $\dfrac{\text{Comp. Force}}{\text{Area}}$ = Pressure]

Compressibility $= \dfrac{1}{K}$

Relationship Between Bulk Modulus (K) and Pressure (P) for a Gas.

(i) For Isothermal Process

$\cancel{PV = RT}$

$P =$

With the change of pressure and temp. the gas undergoes variation in densities. The relationship between pressure (absolute) specific vol and temperature absolute of a gas is given by the eqn. of state as

$$PV = RT$$

where P = Absolute pressure in N/m^2

V = Specific vol $= \frac{1}{\rho}$

T = Absolute Temperature in °K

ρ = density of a gas.

R = Gas constant

$$P \times \frac{1}{\rho} = RT$$

For Isothermal process, the density changes at constant temperature

$$\therefore \frac{P}{\rho} = \text{constant}$$

or $PV = \text{constant}$ $\left[\frac{1}{\rho} = V,\ V = \text{specific vol},\ \rho = \text{density}\right]$

Differentiating this eqn. as P and V are both variables.

$$P\,dV + V\,dP = 0$$

or $P\,dV = -V\,dP$

or $P = -\frac{V\,dP}{dV}$

Putting this value in eqn 1.10 for bulk modulus

$$K = -\frac{dP}{dV} \times V = P$$

$$\therefore K = P \quad \text{---} \ (1.12)$$

(ii) For Adiabatic process – If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. If no heat is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{p}{\rho^k} = \text{constant}$$

where k = Ratio of specific heat of a gas at constant pressure and constant vol = 1·4 for air

$$\frac{p}{\rho^k} = \text{constant}$$

or $p\,v^k = \text{constant}$

Differentiating we get

$$p\,d(v^k) + v^k d(p) = 0$$

or $p\,k\,v^{k-1}\,d(v) + v^k\,d(p) = 0$

or $p\,k\,v^{k-1}\,d(v) + v^{k-1+1}\,d(p) = 0$

or $p\,k\,d(v) + v\,d(p) = 0$ [Cancelling v^{k-1} from both]

or $p\,k\,d(v) = -v\,d(p)$

or $p\,k = -\dfrac{v\,d(p)}{d(v)}$

from eqn. 1·10 for Bulk modulus

$$K = -v\frac{dp}{d(v)}$$

or $K = p\,k$ --- (1·13)

K = Bulk modulus

and k = Ratio of specific heats of a gas at constant pressure and constant vol.

Prob: 1·23

Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm² to 130 N/cm². The vol. of liquid decreased by 0·15 percent.

Soln: Initial pr. = 70 N/cm² = 70×10^4 N/m²

Final pr = 130 N/cm² = 130×10^4 N/m²

Increase in pressure dP = $130\times10^4 - 70\times10^4$

= 60×10^4 N/m²

Decrease in Vol = 0·15 % =

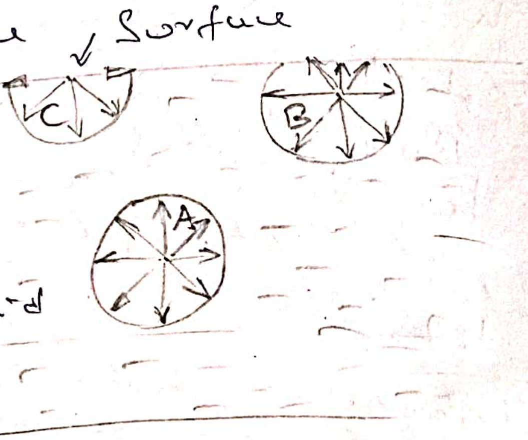
$$\therefore -\frac{dv}{V} = \frac{0.15}{100}$$

$$K = \frac{dP}{-\frac{dV}{V}} = \frac{60 \times 10^4 \, N/m^2}{\frac{0.15}{100}}$$

$$= \frac{60 \times 10^4 \times 100}{0.15} = 4 \times 10^8 \, N/m^2.$$

SURFACE TENSION AND CAPILLARITY

The surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter 'σ' (Sigma). In MKS unit it is expressed as Kgf/m while in S.I unit it is N/m



Consider three molecules A, B and C in a liquid mass. The molecule A is inside the mass

Fig 1.10 Surface Tension

of liquid. Hence this molecule is attracted in all direction equally by ~~molecules~~ surrounding molecules. Hence the resultant force acting on this molecule is zero. The molecule B is near the surface of the liquid. Here the downward force is more than the upward force. Hence the resultant force is ~~downward~~ acting downward. The molecule C which is ~~sitol~~ situated on the surface is acted upon ~~downd~~. downward forces and no upward forces are acting on it. Hence the resultant force is downward force. Thus all the molecules ~~forces~~ on the free surfaces of the liquid experience a downward force. Thus ~~a thin~~ the free surface of the liquid acts like a thin ~~free surface~~ membrane or film under tension ~~of the surface~~ and it acts like an elastic membrane.

The surface tension of a liquid is same ~~at~~ at all points on the liquid surface. However it is affected by temperature. It decreases with increase of temperature.

Units of Surface Tension — It is ~~a~~ tensile force per unit width of the surface film. It is expressed in Newtons per meter (N/m)

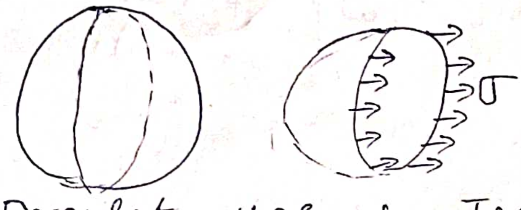
The surface tension for some of the liquids are given below. ~~at~~ at 20°C

Water	0.0728 N/m
Kerosene	0.0277
Glycerine	0.0633
Benzene	0.0289
Mercury	0.5140

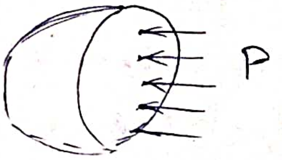
Surface Tension of Water at different temp is given below:

0°C	0.0756 N/m
10°C	0.0742
20°C	0.0728
30°C	0.0712
40°C	0.0696
60°C	0.0662
80°C	0.0626
100°C	0.0589

Pressure Intensity Inside a Droplet



(a) Droplet (b) Surface Tension



(c) Pressure Force

Consider a spherical droplet of a liquid of radius r (or dia d). On the entire surface of the liquid the surface tension will be acting.

Let σ = Surface Tension of the liquid.

P = Pressure intensity inside the droplet (in excess of outside pressure)

d = dia of droplet.

Let us cut the droplet in to two parts and consider one half say left part. The forces acting on this half will be the surface tension it will act on the circumference.

$\therefore$ Tensile force due to surface tension

$= \sigma \times$ circumference $= \sigma \times \pi d$

The pressure force on the cross sectional area will be equal to $P \times \frac{\pi}{4} d^2$

These two forces will be equal under equilibrium condition

$$\therefore \quad P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

Or $$P = \frac{4\sigma}{d} \quad ---- \quad (1.14)$$

Eqn. (1.14) shows that with decrease of diameter of the droplet the pressure inside the droplet increases.

Hollow Bubble

A hollow bubble like a soap bubble has two surfaces in contact with air. One inside the bubble and another outside the bubble. Thus two surfaces are subjected to surface tension.

$$\text{Pressure Force} = p \times \frac{\pi}{4} d^2$$

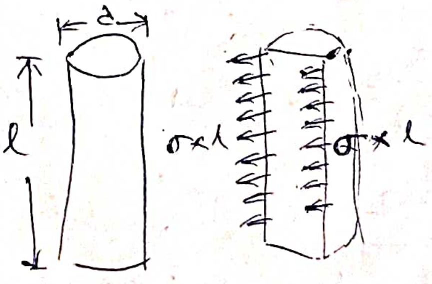
$$\text{Surface Tension} = 2 \times (\sigma \times \pi d)$$

$$p \times \frac{\pi}{4} d^2 = 2 \times \sigma \times \pi d$$

$$p = \frac{8\sigma}{d} \quad ---- \quad (1.15)$$

Surface Tension of a Liquid Jet

Consider a liquid jet of diameter d and length l



Let p = Pressure inside the jet in excess of outside pressure

σ = Surface tension of the liquid.

Let us cut the jet into two parts along the length.

Pressure force acting $= p \times l \times d$ (l×d is area of semi jet / half of jet)

Surface Tension $= \sigma \times l \times 2$

For equilibrium these two should be equal.

$$\therefore \quad p \times l \times d = \sigma \times l \times 2$$

$$\text{or} \quad p = \frac{2\sigma}{d} \quad \text{----} (1.16)$$ or

Prob: 1.25- The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside the droplet of water is to be 0.02 N/cm² greater than the outside pressure. Calculate the diameter of the droplet of water.

SOLN:

$$P = \frac{4\sigma}{d}$$

$$\sigma = 0.0725 \text{ N/m}$$

$$P = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \frac{N}{m^2}$$

$$0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$$

$$\text{or} \quad d = \frac{4 \times 0.0725}{0.02 \times 10^4} = 1.45 \times 10^{-3} \text{ mtr}$$

$$= 0.00145 \text{ mtr.} = 1.45 \text{ mm}$$

Prob 1.26: Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is $\cancel{2.25}$ 2.5 N/m² above atmospheric pressure.

SOLN:

$$P = \frac{8\sigma}{d}$$

$$d = 40 \text{ mm} = 0.04 \text{ mtr}$$

$$P = \cancel{2.25}\ 2.5 \text{ N/m}^2$$

$$\sigma = \frac{p \times d}{8} = \frac{\overset{2.5}{\cancel{2.25}} \times 0.04}{8}$$

$$= \cancel{6.25 \times 10^{-3} \ N/m}$$

$$= \cancel{0.01125 \ N/m}$$

$$= 0.0125 \ N/m$$

Prob: 1.27: The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmosphere pressure). Calculate the pressure with in the droplet if surface tension is given as 0.0725 N/m of water.

Soln: Pressure inside the droplet in excess of outside pressure is given as

$$p = \frac{4\sigma}{d}, \quad \sigma = 0.0725 \ N/m$$

$$d = 0.04 \ mm = 4 \times 10^{-5} \ metre$$

$$\text{Atmospheric Pr} = 10.32 \ N/cm^2 = 10.32 \times 10^{4} \ N/m^2$$

$$p = \frac{4 \times 0.0725}{4 \times 10^{-5}} = 7250 \ N/m^2 = \frac{7250}{10^4} \ N/cm^2$$

$$= 0.725 \ N/cm^2$$

Pressure inside the droplet = p + Outside pr.

$$= 0.725 \ N/cm^2 + 10.32 \ N/cm^2$$

$$= 11.045 \ N/cm^2$$

1.6.4 CAPILLARITY:

Capillarity is defined as the phenomenon of rise or fall of liquid surface in a small tube relative to the adjacent

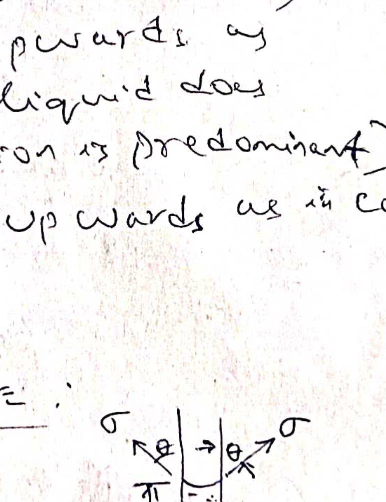
general level of liquid when the tube ~~is~~ held vertically in the liquid.
The rise of liquid surface is known as Capillary rise while the fall of the liquid surface is known as Capillary depression. It is expressed in cm or mm of liquid.

If a glass tube of small diameter ~~is~~ ~~dipped~~. Open at both end is dipped in to a liquid like water, the liquid rises in the tube. On the other hand ~~if~~ a glass tube is dipped in a liquid like mercury, the level of liquid in the tube will be lower than general level of liquid out side.

The above phenomenon is known as Capillarity. It is caused due to ~~phenomenon~~ ~~of~~ the properties of cohesion and adhesion.
~~If the liquid wet the tube~~ The surface of liquid inside the tube is called MENISCUS. If the liquid wets the tube (i.e adhesion is predominant) the meniscus is concave upwards as in case of water. If the liquid does not wet the tube (i.e cohesion is predominant) the meniscus is convex upwards as in case of mercury.

EXPRESSION FOR CAPILLARY RISE:

Consider a glass tube of small diameter 'd', Opened at both ends and is inserted in a liquid, say water.



The liquid will rise in the tube above the level of liquid outside.

Let h = height of the liquid in the tube

Weight of liquid of height h in the tube

$= $ Area of tube $\times h \times \rho \times g = \frac{\pi}{4} d^2 \times h \times \rho \times g$

Let σ = Surface tension at surface of tube

θ = Angle of contact between liquid and glass tube

Surface tension $= \sigma \times \pi d \times \cancel{\cos\theta}$

Component of surface tension acting vertically upward $= \sigma \times \pi d \cos\theta$

The weight of liquid in tube is ~~balle~~ balanced by the the surface tension.

$\therefore \frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \cos\theta$

g = Accela. due to gravity

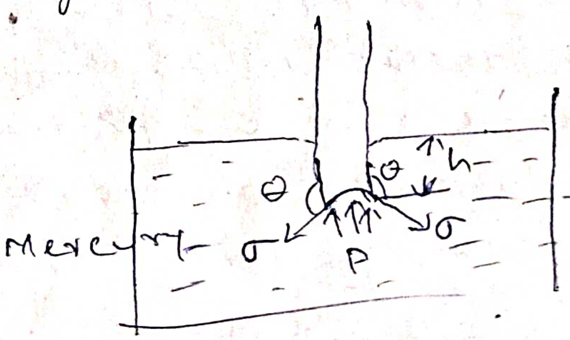
ρ = mass density,

$$h = \frac{\sigma \pi d \cos\theta}{\frac{\pi}{4} \times d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho g d}$$

The value of θ between the clean glass tube and water surface in the tube is approximately equal to zero, Hence $\cos\theta$ is equal to unity.

Thus $h = \frac{4\sigma}{\rho g d}$

CAPILLARY FALL



Surface tension σ

If the glass tube of dia d is dipped in mercury, the level of mercury in the tube will be lower than the general level of mercury out side and the ~~me~~ meniscus will be a covex upword.

Let h = height of ~~liquid~~ depression in tube

θ = Angle of contact between the glass and the ~~liq~~ liquid.

Two forces are acting inside the tube on the mercury

(1) vertical component of The surface tension at the surface of mercury inside the tube $= \sigma \pi d \cos\theta$ acting down wards

(2) The hydrostatic force acting upwards and is equal to intensity of pressure at a depth 'h' × area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \quad (\because p = \rho g h)$$

$$\therefore \quad \sigma \pi d \cos\theta = \rho g h \frac{\pi}{4} d^2$$

$$h = \frac{4 \sigma \cos\theta}{\rho g d}$$

θ for mercury and glass tube is 128°

Prob: 1.29 -

Calculate the capillary effect in millimeters in a glass tube of 4 mm diameter, when immersed in (i) water and (ii) Mercury. The temp. of liquid is $20^\circ C$ and the values of the surface tension of water and mercury at $20^\circ C$ in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130°. Take density of water at $20^\circ C$ as equal to 998 Kg/m^3

Soln

Dia of tube = $d = 4\,mm = 4\times10^{-3}\,m$

The capillary effect i.e. capillary rise or depression is given by eqn $h = \dfrac{4\sigma\cos\theta}{\rho g d}$

σ = Surface tension =

θ = Angle of contact of liquid in tube with glass

ρ = density of liquid.

(i) Capillary rise in water

$\sigma = 0.073575\,N/m$, $\theta = 0°$

$\rho = 998\,kg/m^3$, $d = 4\times10^{-3}\,m$

$\therefore\ h = \dfrac{4\times0.073575\cos0°}{998\times9.81\times4\times10^{-3}} = 7.51\times10^{-3}\,m$

$= 7.51\,mm$

(ii) The capillary rise in mercury

$\sigma = 0.51\,N/m$, $\theta = 130°$

$\rho = 13.6\times1000 = 13600\,kg/m^3$

$h = \dfrac{4\sigma\cos\theta}{\rho g d} = \dfrac{4\times0.51\cos130°}{1360\times9.81\times4\times10^{-3}}$

$= -2.46\times10^{-3}\,m = -2.46\,mm$.

The negative sign indicate the capillary depression.

Prob: 1.31 Find the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m

SOLN:

Capillary rise $h = 2\,mm = 2\times10^{-3}\,m$

$\sigma = 0.073575\,N/m$

d = dia of tube

The angle of θ for water = 0

$$\therefore h = \frac{4\sigma\cos\theta}{\rho g d} = \frac{4 \times 0.073575 \cos 0^\circ}{1000 \times 9.81 \times d}$$

$$2 \times 10^{-3} = \frac{4 \times 0.073575 \cos 0^\circ}{1000 \times 9.81 \times d}$$

$$d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m}$$

$= 1.5$ cm. Ans.

VAPOUR PRESSURE AND CAVITATION

A change from liquid state to gaseous state of a fluid is known as Vaporization. This Vapourization depends on prevailing pressure and temperature. The Vapourization occurs because of continuous escaping of the molecules through the free liquid surface.

Consider a liquid is confined in a closed vessel. Let the temperature of the liquid is 20°c and pressure is atmospheric. This liquid will vaporise at 100°c. When Vapourization takes place the molecules escape from surface of liquid and accumulate in the ~~spa~~ space between liquid surface and top of the vessel. This accumulated ~~pressure~~ Vapour exert pressure on the liquid surface and it is known as VAPOUR pressure of the liquid. Or this is the pressure at which the liquid is converted to Vapour.

Again consider the same liquid at 20°C at atmospheric pressure in the closed vessel. If the pressure above the liquid surface is reduced by some means, the boiling temperature will also be reduced. If the pressure is reduced equal to or below vapour pressure, the boiling of the liquid will start, though the temperature is only 20°C. Thus a liquid may boil even at ordinary temp. even if the pressure above the liquid surface is reduced to equal or less than vapour pressure.

Suppose a liquid is flowing inside a system. If pressure at any point during this flow become equal to or less than the liquid vapour pressure, the vapourization of liquid starts. The bubbles of these vapour flow along with liquid in to the region of high pressure. And here the bubbles collapse giving rise to high ~~pressure~~ impact pressure. The pressure developed by the collapsing bubble is so high that the material from the adjoining material gets eroded and cavities are formed on them. This phenomenon is known as <u>cavitation</u>.

<u>COHESION AND ADHESION</u>

<u>Cohesion</u> is a property of a liquid by virtue of which the molecules of the liquid remain attached to each other. It is due to cohesion a liquid remains a continuous mass rather than individual segregated fragments.

<u>Adhesion</u> is a property of liquid by virtue of which it is enabled to adhere to another body with which it comes

into contact. It is due to the property of adhesion a liquid wets a body which comes in contact ~~of liquid~~ with it.

For example, if a rod is dipped in water and then taken out, the rod becomes wet since some water molecules adhere to the rod.

Mercury is absolutely cohesive but does not adhere to any external object.

PRESSURE AND ITS MEASUREMENT

Fluid Pressure at a Point 2·1

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface. Let dF be the force acting on the area dA, then $\frac{dF}{dA}$ is known as intensity of pressure or pressure acting on area dA. Mathematically it is represented by

$$P = \frac{dF}{dA} \quad \text{where } P = \text{Pressure}.$$

If Force F is uniformly distributed on surface A, then pressure at any point is given as

$$P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}, \quad \text{Hence Pressure Force, } F = P \times A$$

Unit of pressure is M.K.S — Kgf/m^2
S.I — N/m^2 = Pascal = Pa
CGS = $Dyne/cm^2$

(KPa) Kilopascal = 1000 pascal = 1000 N/m^2

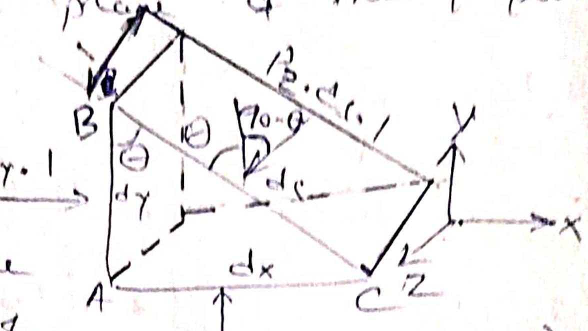
1 bar = 100 KPa = 100×1000 Pascal
= ~~100~~ 10^5 Pascal = $10^5 N/m^2$

PASCAL'S LAW (2·2)

The intensity of pressure ~~intensity~~ or or pressure at a point in a static fluid is equal in all direction.

Proof: Consider an arbitrary fluid element of wedge shape of size dx×dy×ds as shown in figure 2·1. Let the width

If the element be unit which is perpendicular to the plane of the paper in Z axis.



(FIG 2.1)

P_x, P_y and P_z are the pressure or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are

1. Pressure force normal to the surfaces
2. Weight of the element in the vertical direction.

Forces acting on the surfaces are

On face AB $= P_x \times dy \times 1$ ($dy \times 1$ = area of face AB)

On face AC $= P_y \times dx \times 1$

On face ds $=$
$P_z \times ds \times 1$

Weight of the element = Vol × density × g

$= \frac{1}{2} \times AB \times AC \times 1 \times \rho \times g = \frac{1}{2} \times dy \times dx \times \rho \times g$

where ρ = density of element
g = Acceln due to gravity

Projecting the forces on horizontal axis (x-axis)

$P_x . dy . 1 - P_z . ds . 1 \sin(90-\theta) = 0$

or $P_x . dy - P_z . ds . \cos\theta = 0$

From Δ ABC, $ds \cos\theta = dy$ ($BC \cos\theta = AB$)

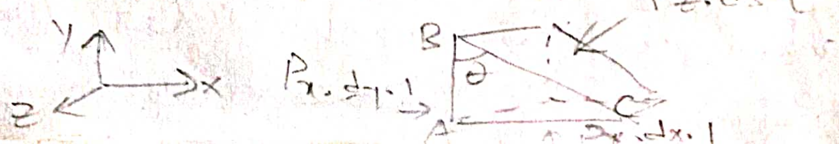
$\therefore P_x\, dy - P_z . dy = 0$

or $P_x = P_z$. ------ (2.1)

Now projecting the forces on vertical 'y' axis

$P_y . dx . 1 - P_z . ds . 1 \cos(90-\theta)$
$- \frac{1}{2} . dy . dx \times \rho \times g = 0$

or



or $P_y \cdot dx - P_z \cdot ds \cdot \sin\theta - \frac{1}{2} \times dy \cdot dx \cdot \rho \times g = 0$

As the element is very small we can neglect its weight.

Again in Δ ABC, $ds \sin\theta = dx$

$\therefore \quad P_y \cdot dx - P_z \cdot dx = 0$

or $\quad P_y = P_z \quad ---- \quad (2.2)$

$\therefore$ from eqn. 2.1 and 2.2

$P_x = P_y = P_z$.

Which shows that pressure at any point in a fluid is equal in x, y, and z direction. Since the choice of fluid element was arbitrary hence pressure at any point is same in all direction.

2.3 PRESSURE VARIATION IN A FLUID AT REST

HYDROSTATIC LAW – The rate of increase of pressure in a vertically downward direction is equal to the specific weight of the fluid at that point.

Consider a small element of cross sectional area ΔA as shown in fig 2.2

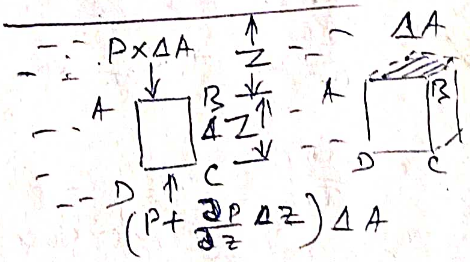


FIG. 2.2

ΔA = Cross section of Area

ΔZ = Height of the element

Let Z = distance of the element from surface of water

The forces acting of the elements are

Pressure force on its surfaces

and weight of the element

Let p = pressure ~~of~~ acting on face AB at a ~~at~~ depth of Z from surface of water.

Force of face AB = $p \times \Delta A$ (ΔA is the C.S. of element)

This force will act downward.

Force acting on face CD

$$= \left(p + \frac{\partial p}{\partial z} \times \Delta z\right) \times \Delta A$$

$\frac{\partial p}{\partial z}$ = Rate of pressure variation with depth

Δz = height of the element

This force will act upward on face CD

Weight of element = Vol × ρ × g

$= \Delta A \times \Delta z \times \rho \times g$ (This will act downward)

ρ = Density

g = Accln due to gravity

∴ For equilibrium

$$p \times \Delta A - \left(p + \frac{\partial p}{\partial z} \times \Delta z\right) \times \Delta A + \Delta A \times \Delta z \times \rho \times g = 0$$

or $$p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial z} \times \Delta z \, \Delta A + \Delta A \times \Delta z \times \rho \times g = 0$$

or $$-\frac{\partial p}{\partial z} \Delta A \, \Delta z + \Delta A \, \Delta z \, \rho g = 0$$

or $$\frac{\partial p}{\partial z} = \rho g = w \quad (w = \text{Specific wt} = \rho \times g)$$

∴ $$\frac{\partial p}{\partial z} = w \quad \text{-----} \quad (2.4)$$

The rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point.

This is Hydrostatic Law.

From eqn (2.4)

$$\partial p = w \, \partial z$$

Integrating both sides.

$P = \cancel{wz}$

$\int \partial p = \int w \, \partial z$

or $p = wz = \rho g z$ ---- (2·5)

or $Z = \dfrac{p}{\rho g}$ ---- (2·6)

Z = Pressure Head.

Prob: 2.1 : A hydraulic press has a ram of 30 cm diameter and a plunger of 4·5 cm diameter. Find the weight lifted by the hydraulic press when the force applied to the plunger is 500 N

SOLN :

Dia of Ram = 30 cm = 0·3 m

C-S. Area of Ram $= \frac{\pi}{4} d^2 = \frac{\pi}{4}(0{\cdot}3)^2 = 0{\cdot}0706858 \ m^2$

Dia of plunger = 4·5 cm = 0·045 m

C-S. Area of plunger $= \frac{\pi}{4} d^2 = \frac{\pi}{4}(0{\cdot}045)^2$

$= 1{\cdot}5904 \times 10^{-3} \ m^2$

Force applied at plunger = 500 N

∴ Pressure Intensity at surface of plunger

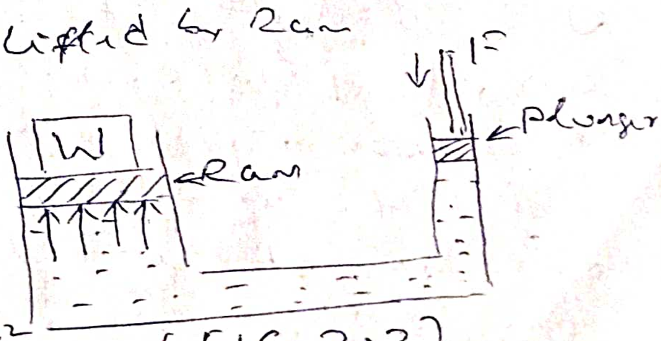
$= \dfrac{500}{1{\cdot}5904 \times 10^{-3}} \ N/m^2$

This pressure Intensity is transmitted to Ram.

Let W is the Weight lifted by Ram

∴ Force acting at Ram = W

Pressure $= \dfrac{W}{A}$



(FIG-2·3)

∴ $\dfrac{500 \ N}{1{\cdot}5904 \times 10^{-3} \ m^2} = \dfrac{W}{0{\cdot}0706858 \ m^2}$

or $W = 22222{\cdot}648 \ N = 22{\cdot}222648 \ kN$

Prob: 2.5 An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.

SOLN:

Sp. gr. of Oil $= 0.9 = S_o$

Height of Oil $= 40\ m = Z_o$

Density of oil ρ_o = Sp. gr. of oil × Density of water

$= 0.9 \times 1000\ kg/m^3 = 900\ kg/m^3$

Intensity of pressure $P = \rho_o \times g \times Z_o$

$= 900 \times 9.81 \times 40 = \overset{353160}{\cancel{353.160}}\ N/m^2$

To Calculate Corresponding Height of Water :

$P = \rho_w \times g \times h_w$ $\quad \rho_w = 1000\ kg/m^3$

$P = 353.160\ N/m^2$

$g = 9.81\ m/sec^2$

$\therefore h_w = \dfrac{\overset{353160}{\cancel{353.160}}}{1000 \times 9.81} = \cancel{0.036\ m}$

$= 36$ mtr (of water.)

PROB. 2.6 : An open tank contains water upto a depth of 2 mtr and above it an oil of sp. gravity 0.9 for a depth of 1 ~~mtr~~ m. Find the pressure intensity (i) at the interface of the two liquids and (ii) at the bottom of the tank.

SOLN :

Height of water $Z_1 = 2.0$ mtr

Height of Oil $Z_2 = 1.0$ metre.

Sp. Gr. of Oil $= 0.9$

$\therefore$ Density of Oil $=$

$\rho_o = 0.9 \times 1000 = 900\ kg/m^3$

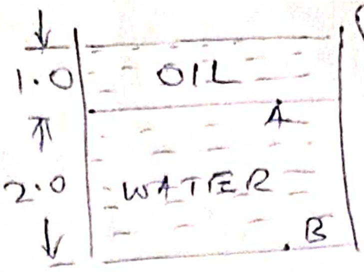


FIG - 2.4

(i) Pressure at Interface of two liquids i.e at A

$\Rightarrow P = \rho g h = \rho_o \times 9.81 \times 1$ metre

$= 900 \times 9.81 \times 1 = 8829\ N/m^2$

(ii) Pressure at bottom of the tank B

$\Rightarrow P = \rho_o g h_o + \rho_w g h_w$

$= 900 \times 9.81 \times 1 + 1000 \times 9.81 \times 2.0$

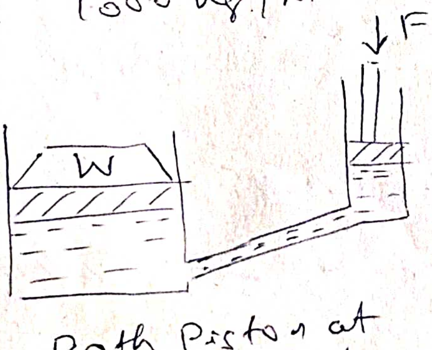
$= 8829\ N/m^2 + 19620\ N/m^2 = 28449\ N/m^2$

PROB 2.7 The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when

(a) the pistons are at the same level.

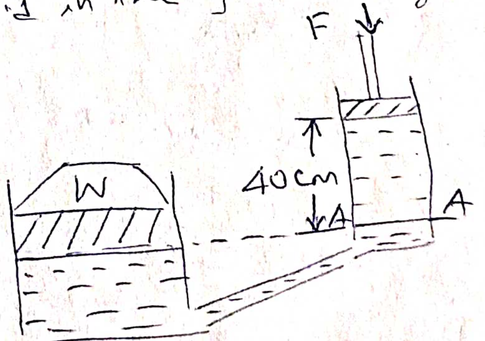
(b) Small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as $1000\ kg/m^2$



Both Piston at Same Level

(FIG - 2.5)



SMALL PISTON IS 40 CM ABOVE LARGE PISTON

(FIG - 2.6)

SOLN

Dia of Small piston = d = 3 cm = 0.03 m

Area of Small piston = $(0.03)^2 m^2 \times \frac{\pi}{4}$

$= 7.06858 \times 10^{-4}\ m^2$

Dia of Large piston = D = 10 cm = 0.1 mtr

Area of Large Piston = $\frac{\pi}{4} D^2 = \frac{\pi}{4} \times (0.1)^2$

$= 7.8539 \times 10^{-3}\ m^2$

Let the Load lifted = W

(a) When both the pistons are at the same level (FIG 2.5)

Pressure intensity on the small piston

$= \frac{F}{Area} = \frac{80}{7.06858 \times 10^{-4}} = 113176.904\ N/m^2$

This ~~force~~ pressure is transmitted to large piston:

∴ Force on Large Piston = Pressure × Area

$= 113176.904 \times 7.8539 \times 10^{-3}\ m^2$

= 888.880 N (Load lifted by large piston)

(b) When the small piston is 40 cm above the large piston (FIG-2.6):

Pressure intensity due to force of 80 N

$= \frac{80}{7.06858 \times 10^{-4}} = 113176.904\ N/m^2$

Pressure due to height of 40 cm of liquid

$= \rho g h = 1000 \times 9.81 \times \frac{40}{100} = 3924\ N/m^2$

Total pressure intensity = 113176.904 + 3924

= 117100.904 N/m²

$\therefore$ Pressure intensity transmitted to large piston

$117100 \cdot 904 \ N/m^2$

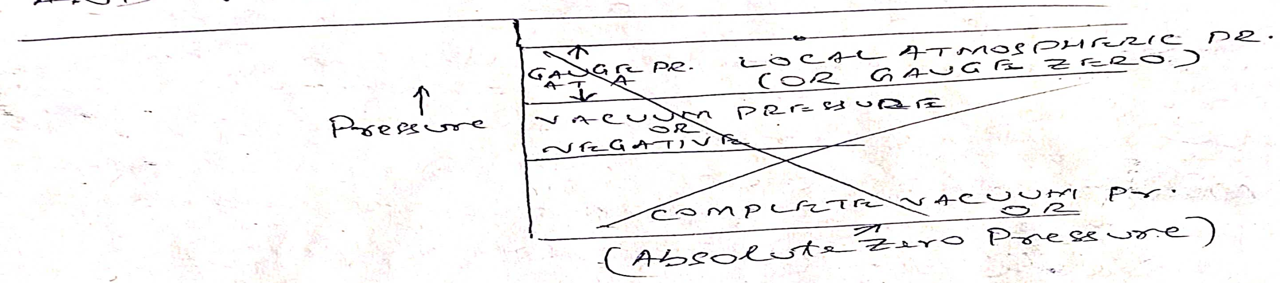
Area of large piston $= 7 \cdot 8539 \times 10^{-3} \ m^2$

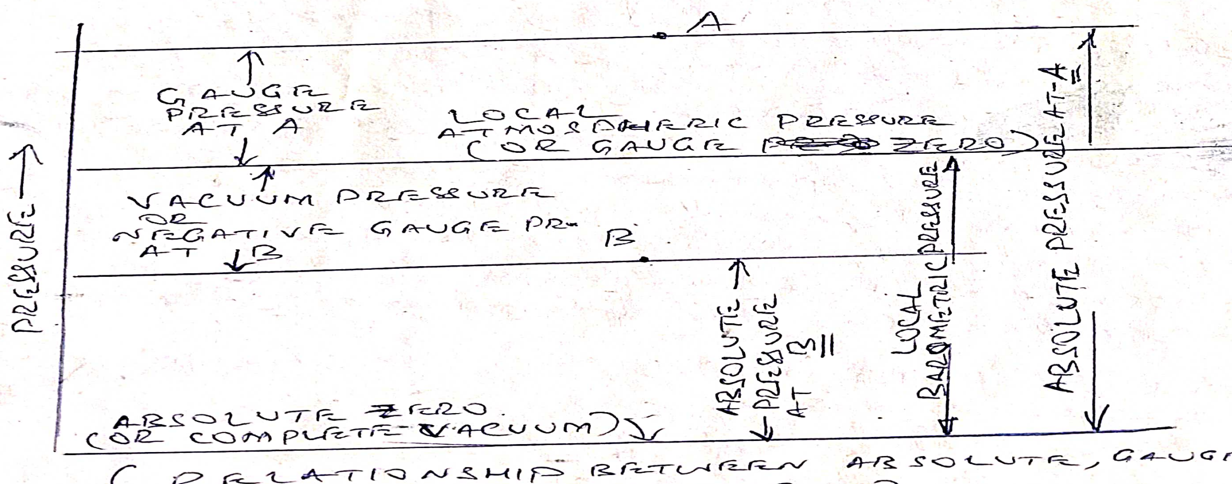
$\therefore$ Force on large piston = Pressure × Area

$= 117100 \cdot 904 \ N/m^2 \times 7 \cdot 8539 \times 10^{-3} \ m^2 =$

$= 919 \cdot 6987 \ N$ (Load lifted by large piston)

2·4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURE





(RELATIONSHIP BETWEEN ABSOLUTE, GAUGE AND VACUUM PRESSURE)

ATMOSPHERIC PRESSURE

The atmosphere air exerts normal pressure on all surfaces with which it is in contact, and it is known as atmosphere pressure. The atmospheric pressure varies with altitude and it can be measured by means of a barometer. Hence it is also called barometric pressure. At sea level under normal condition ($15^\circ C$) the value of atmospheric pressure is $1.01043 \times 10^4 \, N/m^2$, or $1.033 \, kg(f)/cm^2$ or 10.33 m of water or 760 mm of mercury.

Fluid pressure are ~~may be~~ measured with respect to two datum. They are (i) Absolute zero pressure and (ii) Local Atmosphere pressure.

ABSOLUTE PRESSURE - It is defined as the pressure which is measured above absolute zero (or complete vacuum).

GAUGE PRESSURE - When the pressure is measured either above or below atmosphere pressure, with atmosphere pressure as datum it is called gauge pressure. All pressure gauges read zero when opened to atmosphere and read only the difference between the pressure of the fluid to which they are connected and the atmosphere pressure.

VACUUM PRESSURE - It is the pressure below the atmospheric pressure. It is also called suction pressure or negative gauge pressure and its ~~gage~~ gauge value is the amount by which it is below that of atmospheric pressure.

All the values of absolute pressure are positive, since in the case of fluids the lowest absolute pressure which can possibly exist corresponds to absolute zero or complete vacuum. However gauge pressure are positive if they are above atmosphere and negative if they are below atmosphere (vacuum pr.)

(i) Absolute pressure = Atmospheric pressure + Gauge pressure

$$P_{abs} = P_{atm} + P_{gauge}$$

(ii) Vacuum Pressure = Atmospheric Pressure − Absolute pressure.

Prob: 2.8
What are the gauge pressure ~~as~~ and absolute pressure at a point 3 M below the free surface of a liquid having a density of 1.53×10^3 Kg/m³ if the atmospheric pressure is equivalent to 750 mm of mercury? The specific gravity of mercury is 13.6 and density of water = 1000 Kg/m³

SOLN Depth of Liquid = 3 M = Z_1

Density of Liquid = 1.53×10^3 Kg/m³

Atmospheric Pressure Head Z_0 = 750 mm of Hg = 0.75 mtr of Hg

Atmospheric Pr. $P_{atm} = \rho_0 g h_0 = \rho_0 g Z_0$

ρ_0 = density of mercury = sp. gr. of mercury × density of water

$= 13.6 \times 1000 \text{ Kg/m}^3 = 13600 \text{ Kg/m}^3$

$\therefore P_{atm} = 13600 \times 9.81 \times 0.75 = 100062 \text{ N/m}^2$

Pressure at a point 3 Mtr ~~from~~ below the free surface of Liquid $= \rho_1 g z_1 = P_1$

ρ_1 = Density of Liquid $= 1.53\times10^3 kg/m^3$

$g = 9.81\ m/sec^2$

$z_1 = 3\ m$

$\therefore\ P_1 = 1.53\times10^3 \times 9.81\times 3 = 45027.9\ N/m^2$

This is the gauge pressure.

$\therefore$ Absolute pressure = Atmospheric pr + gauge pr.

$= 100062\ N/m^2 + 45027.9\ N/m^2$

$= 145089.9\ N/m^2$

2.5

MEASUREMENT OF PRESSURE

The following devices are used to measure pressure of a fluid

1. Manometer 2. Mechanical Gauges.

~~Ex~~

2.5.1 MANOMETER – ~~Ex~~ A manometer is a device used to measure the pressure at a point in a fluid by balancing the column of fluid by the same fluid, or another column of fluid. They are classified as

(a) SIMPLE Manometer

(b) DIFFERENTIAL Manometer.

BAROMETER: It is a device used for measuring the local atmospheric pressure. It consist of a container partly filled with mercury and a small diameter (3 to 5 mm) glass tube of 100 cm length which is closed at one end.

First fill the glass tube completely with mercury and then dip into the container and keep it vertically. A small quantity of mercury will drop in to the container and vacuum forms at the upper end of the tube.

The atmospheric pressure on the surface of the mercury container will support a mercury column in the tube which is represented by a head ~~H cm of~~ H cm of mercury

$$P_{atm} = w_{Hg} H \qquad w = \text{Specific wt. of Mercury} = \rho g = 13600 \text{ kg/m}^3$$

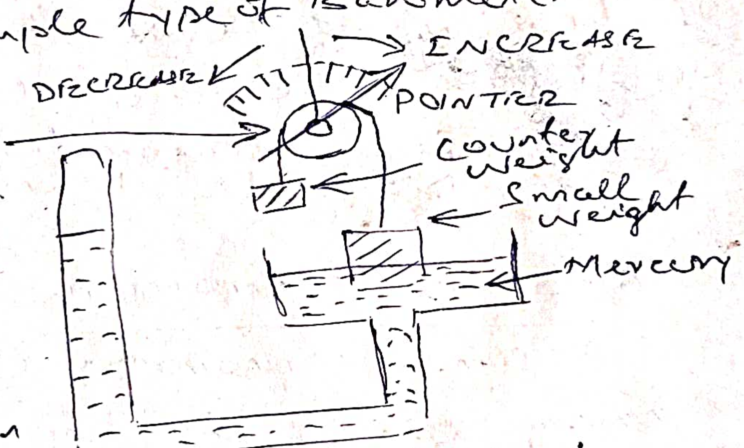
$$\text{or } H = \frac{P_{atm}}{w_{Hg}}$$

(Atmospheric Pressure Head in terms of Hg)

At sea level $H = 76$ cm and it changes from place to place as per height above sea level.

SIPHON BAROMETER

This is another simple type of Barometer.



It consist of a U ~~tube~~ type glass tube which is closed at one end and is enlarged at the other end to form a container. A small iron weight is supported on the mercury surface of the container partly by buoyancy force of mercury (upward) and partly by a small counter weight. The iron weight and the counter weight are connected by a string and it passes over a pulley.

Variation in atmospheric pr. brings rise or fall of the mercury surface in the open end of the U tube which produces the rotation of the pulley by a small angle. This angle is calibrated in terms of Hg height. The pointer attached to the pulley will move over a calibrated circular scale which can directly read the atmospheric pressure.

MANOMETER:

Manometer is divided into two heads

1. Simple Mano Meter
2. Differential Manometer

The pressure in a fluid can be measured by using the following device.

1. Manometer
2. ~~Mechanical G~~ Mechanical Gauges.

MANOMETER: It is a device used to measure the pressure of fluid at a point by balancing the column of fluid, by the ~~th~~ same fluid or different column of fluid. They are classified as

(i) Simple Manometer
(ii) Differential Manometer.

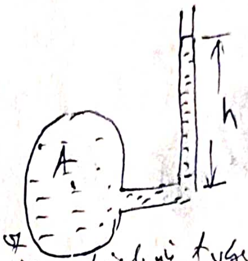
2.6 SIMPLE MANOMETER

A simple manometer consists of a glass tube having one of the ends connected to the point where pressure is to be measured and other end remains open to atmosphere. Common type of simple manometer are

1. Piezometer
2. U-tube - Manometer
3. Single Column Manometer.

2.6.1 PIEZOMETER - It is the ~~sim~~ ~~simplest~~ simplest form of manometer used for measuring gauge pressure. One end of the manometer is connected to the point where pressure is to be measured and the other end is open to the atmosphere as shown in fig 2.8. The rise of liquid gives the pressure head at that point. If at a point A, the height of ~~liq~~ liquid say water is 'h' in piezometer tube, then pressure at A

$= \rho \times g \times h \ N/m^2$

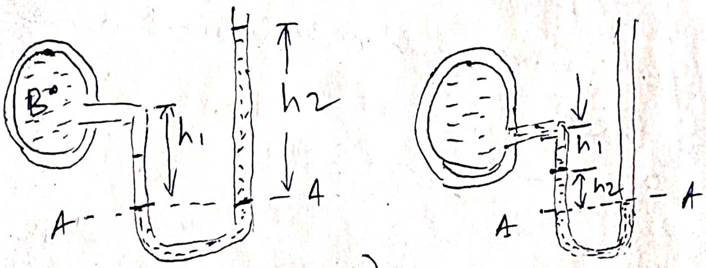


(FIG 2.8)

LIMITATION OF PIEZOMETER:

(i) Difficult to measure high pr. for liquid of low sp. gr., the height of liquid in piezometer will be very high in tube
(ii) It can not work on negative gauge pr. since air would flow in to the container through tube
(iii) Capillary errors are likely to arise when the tubes are of diameter 10 mm or less.
(iv) Piezometer can not cope up with rapid change in pressure.

U TUBE MANOMETER



(a) (FOR GAUGE PRESSURE) (b) (FOR VACUUM PRESSURE)

FIG - 2.9

(a) FOR GAUGE PRESSURE:

Let B be the point ~~wh~~ at which pressure is to be measured. Assume 'p' is the pressure at B

The datum line is A-A

Let h_1 = height of light liquid above datum line whose pressure is to be measured.

h_2 = height of the ~~light~~ heavy liquid above datum line

S_1 = sp. gr. of light liquid

S_2 = sp. gr. of heavy liquid

ρ_1 = Density of light liquid

ρ_2 = Density of heavy liquid

The pressure is same for one horizontal level

$\therefore$ Pressure above datum line A-A in the left and right column of the U tube manometer should be same.

Pressure above AA in left column

$= P + \rho_1 g h_1$

Pressure above AA in the right column

$= \rho_2 g h_2$

$\therefore \quad P + \rho_1 g h_1 = \rho_2 g h_2$

$P = \rho_2 g h_2 - \rho_1 g h_1 \quad ----(2.7)$

(b) FOR VACUUM PRESSURE

For measuring vacuum pressure, the level of ~~the~~ liquid in the manometer will be as shown in fig 2.9 (b)

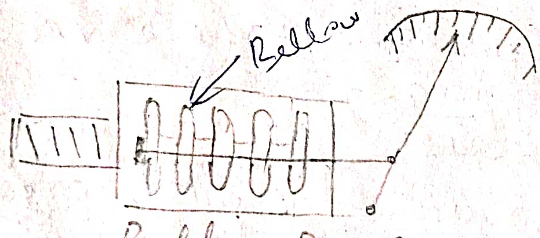
Pressure above AA in left column

$= \rho_2 g h_2 + \rho_1 g h_1 + P$

Pressure above AA in right column $= 0$

$\therefore \quad \rho_2 g h_1 + \rho_1 g h_1 + P = 0$

or $\quad P = -(\rho_1 g h_1 + \rho_2 g h_2) \quad ---(2.8)$



Bellow Pr. Gauge

In this ~~gauge~~ gage the pressure responsive element is made up of a thin metallic tube having deep circumferential corrugation. In response to the pressure changes this elastic element expands or ~~contact~~ contracts, thereby moving the pointer on a graduated circular dial as shown

$$P = \rho g h_e$$

$$h_e = \frac{P}{\rho g}$$

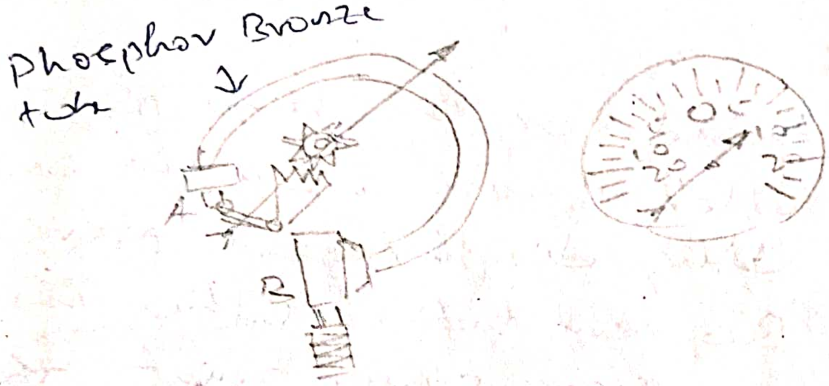
$$Sp.\ gr. = \frac{\text{wt. density of liquid}}{\text{wt. density of water}}$$

$$\rho_m g \times h_m = \rho_w \times g \times h_w$$

$$h_w = \frac{\rho_m \times g \times h_m}{\rho_w \times g}$$

$$= S_m \times h_m$$

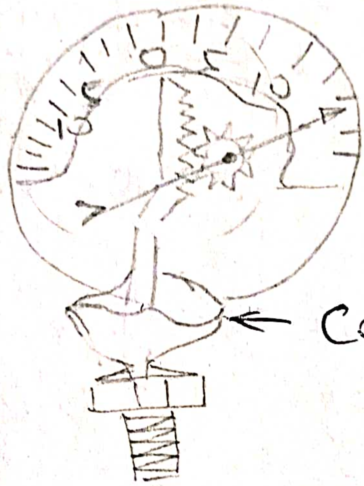
The BOURDON GAUGE



This device consist of a metallic tube of elliptical section closed at one end A. the other end B being fitted to the gauge point where the pressure is to be measured. As the fluid enters the tube, the ~~ends~~ tube tends to straighten. By using a pinion-sector arrangement the small elastic deformation of the tube is transmitted to a pointer in an ~~op~~ amplified manner. The pointer moves over a graduated dial. The device is calibrated by subjecting it to various known pressure.

The bourdon gauge is suitable for measuring not only high pressure such as those in steam boiler or water main but also negative or vacuum pressure. A gauge which is devised to measure positive as well as negative pressure is called a compound gauge.

DIAPHRAGM PRESSURE GAUGE



This device is based on the same principle as that of the Bourdon gauge. In this case a corrugated ~~diag~~ diaphragm is provided instead of the Bourdon tube. When the device is fitted to any gauge point, the diaphragm will undergo an elastic deformation. This deformation is communicated to a pointer which moves on a graduated scale indicating the pressure. It may be noted that this device works on the same principle as that of the aneroid barometer. This device is found suitable for measuring relatively low pressure.

MICRO MANOMETERS